X-ray free-electron lasers:
An introduction to the physics and main characteristics

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Acknowledgements and References

- Lots of great material on the subject.
- The notes for this lectures have been freely inspired by:
  - Z. Huang and K. J. Kim
    - USPAS course. FEL lecture notes, available electronically
  - T. Raubenheimer SSSEPB school
    - https://conf-slac.stanford.edu/sssepb-2013/lectures
  - C. Pellegrini FEL tutorial 2014
    - https://conf-slac.stanford.edu/uxss-2014/sites/conf-slac.stanford.edu.uxss-2014/files/Pellegrini.pdf

Outline

- Introduction: Why an X-ray FEL? What are the key-elements to build one?
- FEL physics I: basic properties of the radiation from accelerated relativistic electrons
- FEL Physics II: the FEL collective instability and X-ray FELs
- State-of-the-art
- Conclusions

Introduction

The interest in the X-ray FEL is motivated by:

- wide tunability range, 10 to 0.1nm
- transverse and longitudinal coherence
- high peak power, tens to hundreds GW
- short pulse length, 1 to 100 fs.

The X-ray FEL is the only instrument that can explore matter at the length **and** time scale typical of atomic and molecular phenomena: Bohr atomic radius, about 1 Å, Bohr period of a valence electron, about 1 fs.

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Wide range of applications

The transverse coherence and large number of photons make possible to image periodic and non-periodic structures at the nanometer and sub-nanometer scale.

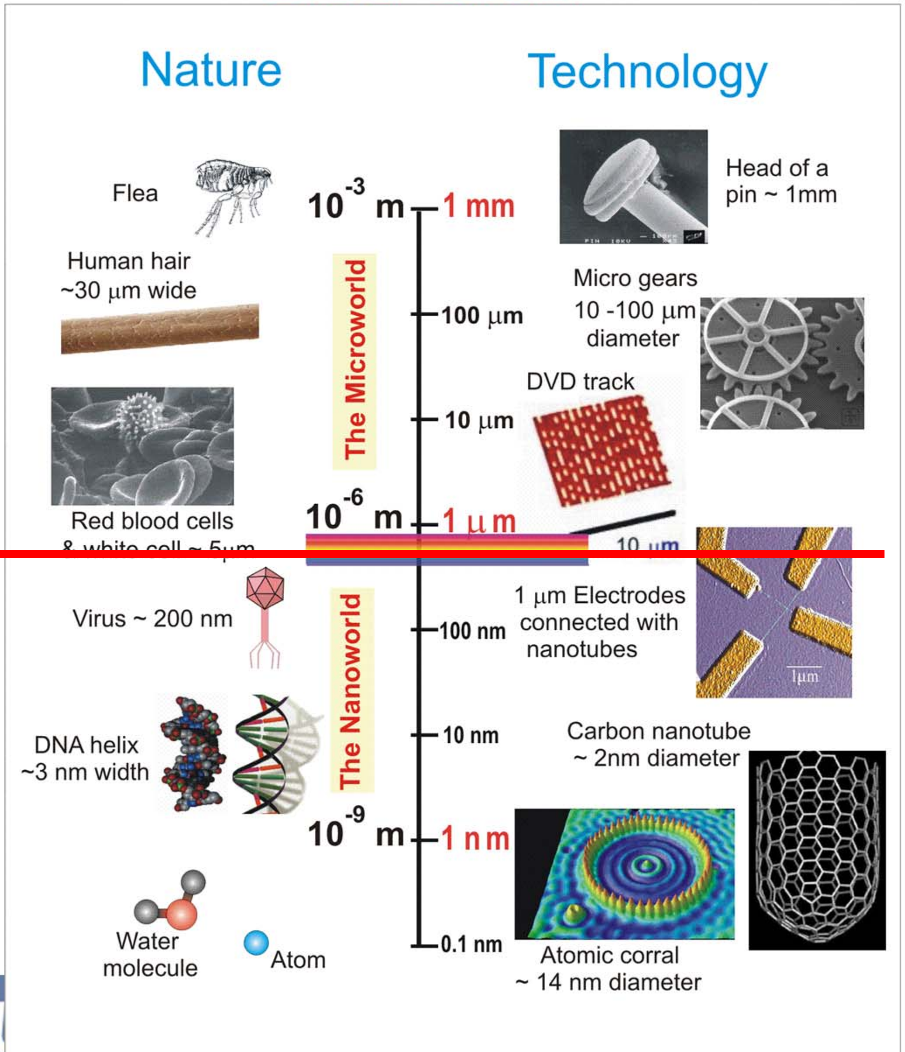
Using fast, single shot imaging, one can follow the dynamics of these phenomena, and overcome the limit set by sample damage.

Non linear phenomena and high energy density systems can be studied using the short pulse duration and large peak power.

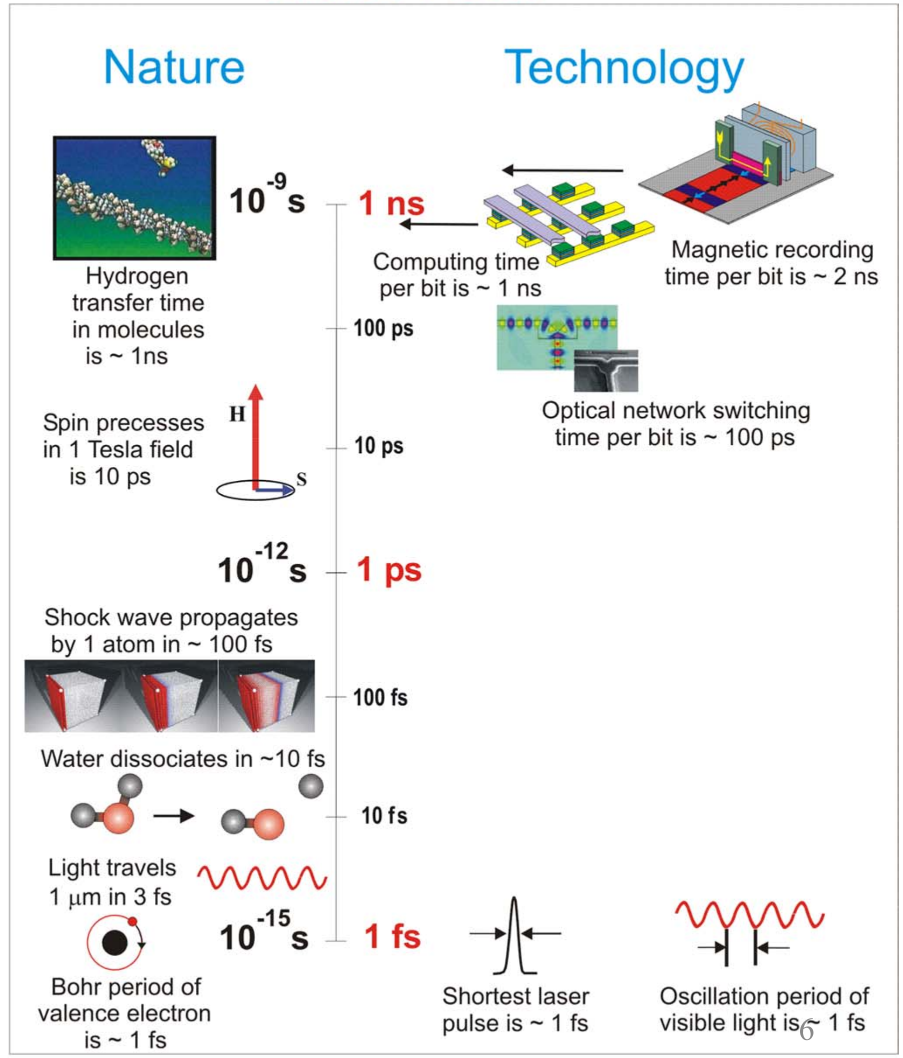
The X-ray FEL can be used to explore matter at atomic molecular scale with unprecedented space-time resolution.

X-Rays have opened the Ultra-Small World
X-FELs open the Ultra-Small and Ultra-Fast Worlds

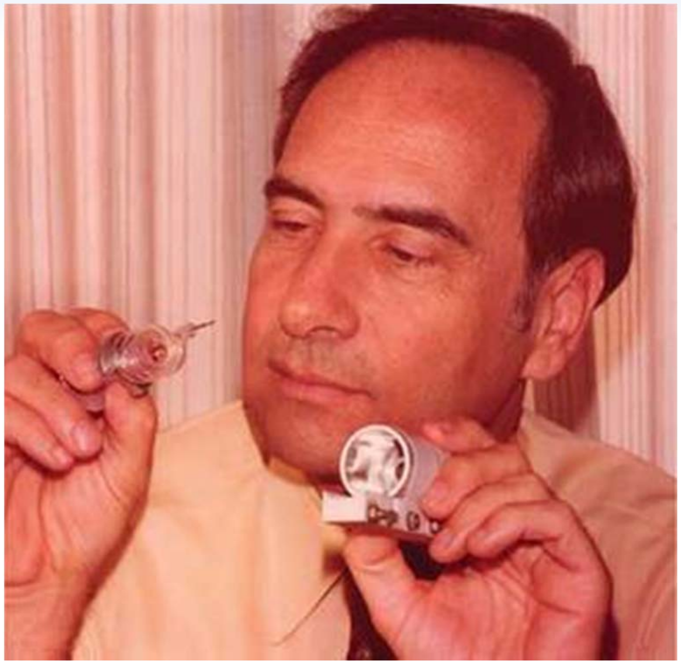
Ultra-Small



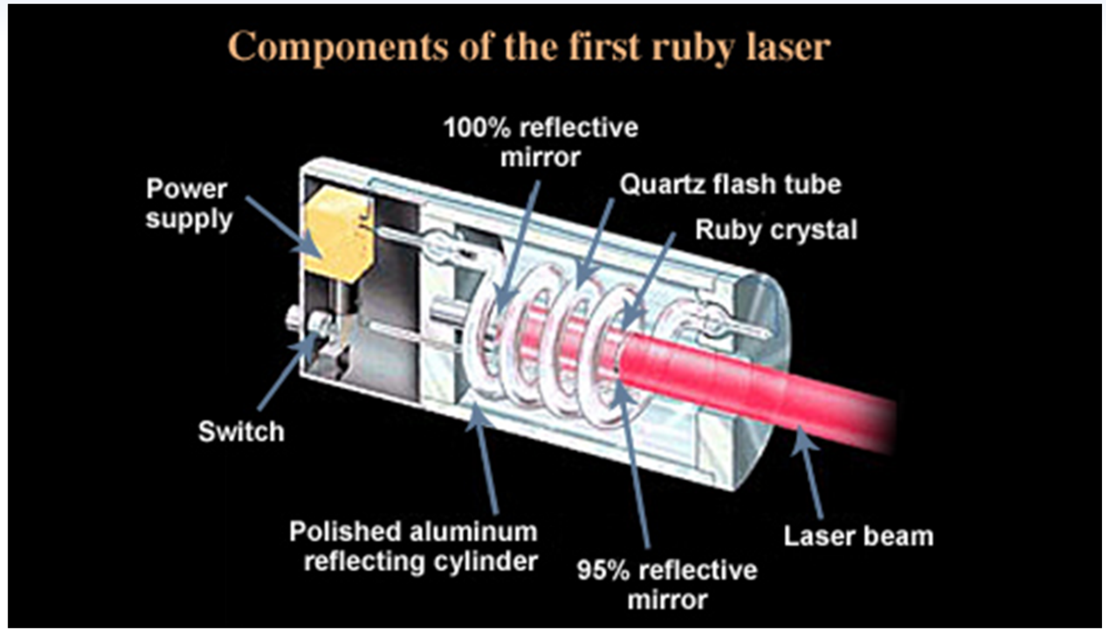
Ultra-Fast



1960: The beginning of lasers



Ted Maiman (25 years after first Ruby laser)



Today lasers cover the spectral region from infrared to EUV with high intensity and short pulses.

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Early work on X-ray lasers

Development of X-Ray lasers has been a major goal almost from the time the first laser was developed in 1960.

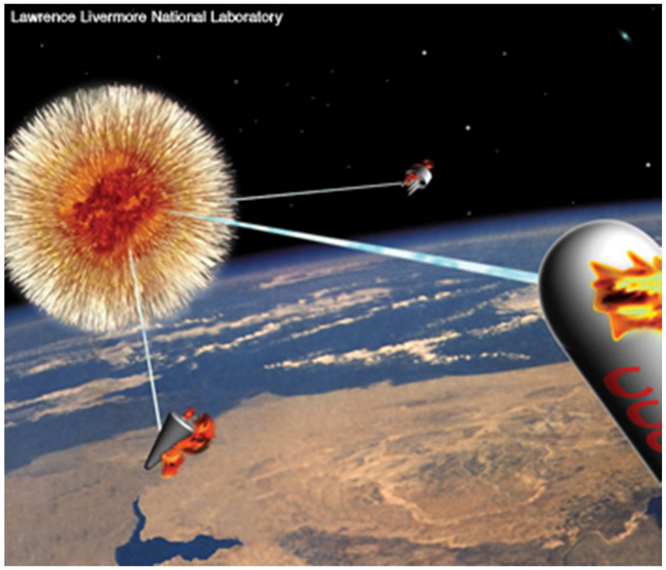
In the conventional atom-based laser approach the task is extremely difficult, because of the very short lifetime of excited atom-core quantum energy levels and of the energy needed to excite inner atomic levels is large, 1 to 10 KeV compared to about 1 eV for visible lasers. As a consequence, a very intense pumping levels is needed to obtain population inversion.

A further complication is the lack of materials able to reflect or transmit light efficiently in the X-ray region, making an X-ray oscillator, very difficult.

Early work on X-ray lasers

To overcome these problems scientists at LLNL proposed to use a nuclear weapon to drive an X-ray laser. They tried the scheme in the Dauphin experiment, apparently with success, in 1980.

The idea, part of Star Wars Defense Initiative, was to generate an X-ray beam in space to kill incoming missiles, exploding an atomic bomb. When Star Wars was terminated the program ended.



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From lasers to Free Electron Lasers

A more convenient solution to the wish for a tunable, high power X-Ray lasers is the SASE (Self Amplified Spontaneous Emission) X-Ray FEL, proposed in 1992, and realized in 2009 when LCLS at Stanford lased at a record short wavelength of 1.5 Angstrom and more recently at SACLA, in Japan.

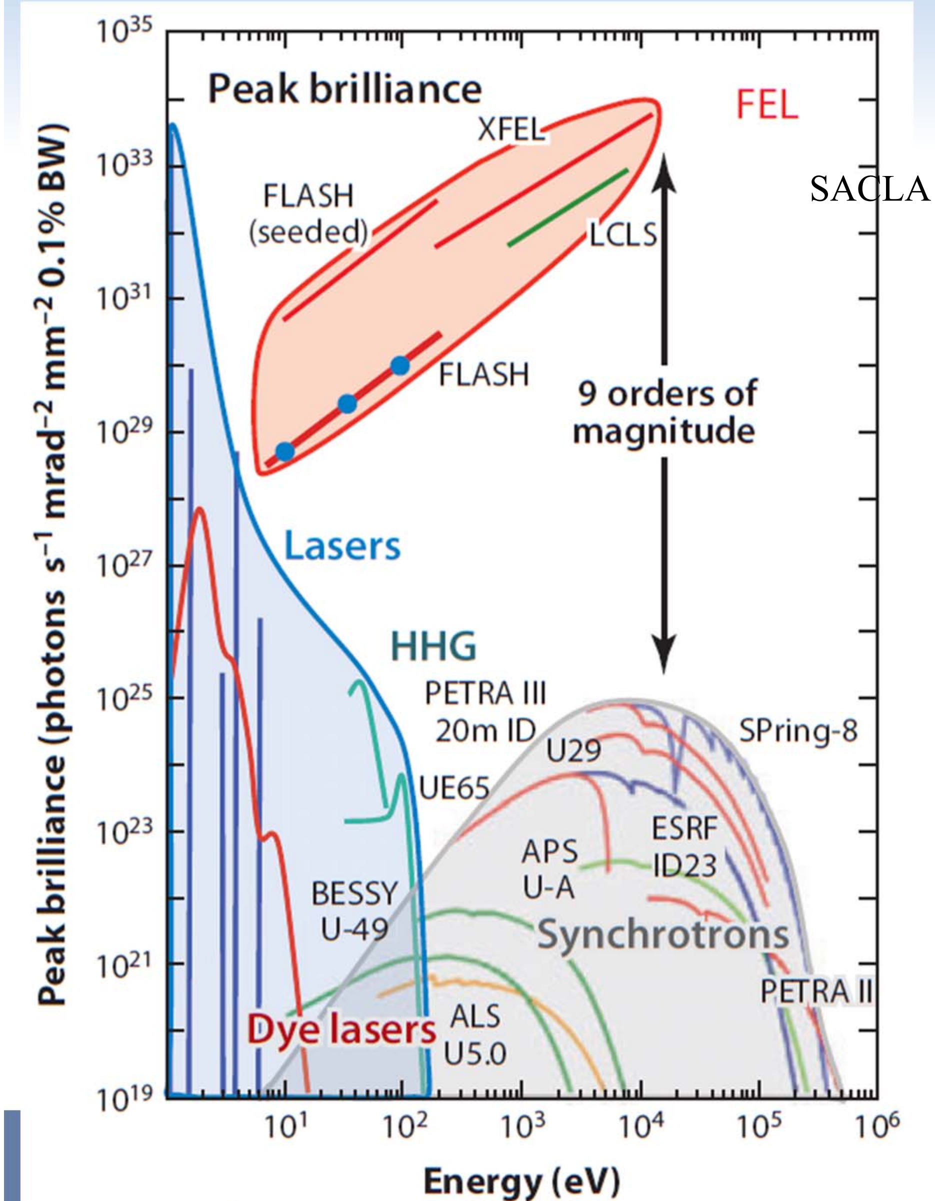
References:

C. Pellegrini, “A 4 to 0.1 nm X-ray FEL based on the SLAC linac”, Proc. of the Workshop on 4th Generation Light Sources, M. Cornacchia and H. Winick eds., SSRL92-02, p. 364-375 (1992).

P. Emma et al., "First lasing and Operation of an angstrom-wavelength free-electron laser", Nature Photonics, 4, 641 (2010).

T. Ishikawa et al., “ A compact X-ray free-electron laser emitting in the sub Ångström region”, Nat. Photon. **6**, 540-544 (2012).

X-FELs and other light sources



Brilliance, also called brightness, is a measure of the coherence and intensity of photon beams. Improved longitudinal coherence will further increase the brilliance.

The jump by 9 orders of magnitude obtained at LCLS in 2009 is a remarkable event.

Plot from J. Ullrich, A. Rudenko, R. Moshammer, Ann. Rev. Phys. Chem. **63**, 635 (2012)

Radiation from free accelerated electrons

When does a free charged particle radiate?

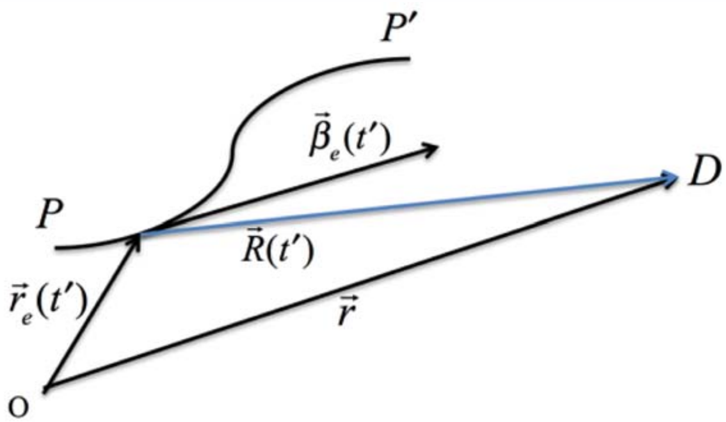
http://www.shintakelab.com/en/enEducationalSoft.htm

1. Accelerated electrons, $\dot{\beta} \neq 0$, synchrotron and undulator radiation;
2. Constant velocity larger than *c/n* in a spatially homogenous medium, Cerenkov radiation
3. Stopping or starting charge, $\dot{\beta}(t) \simeq \delta_f(t)$, transition radiation, edge radiation, bremsstrahlung.
4. Constant velocity for all times in a spatially non homogeneous periodic medium, Smith-Purcell.

The radiation can be broadband or narrow band. All form of radiation are broadband except when a periodicity is imposed so that a narrow band is selected by interference.

Lienard-Wiechert fields

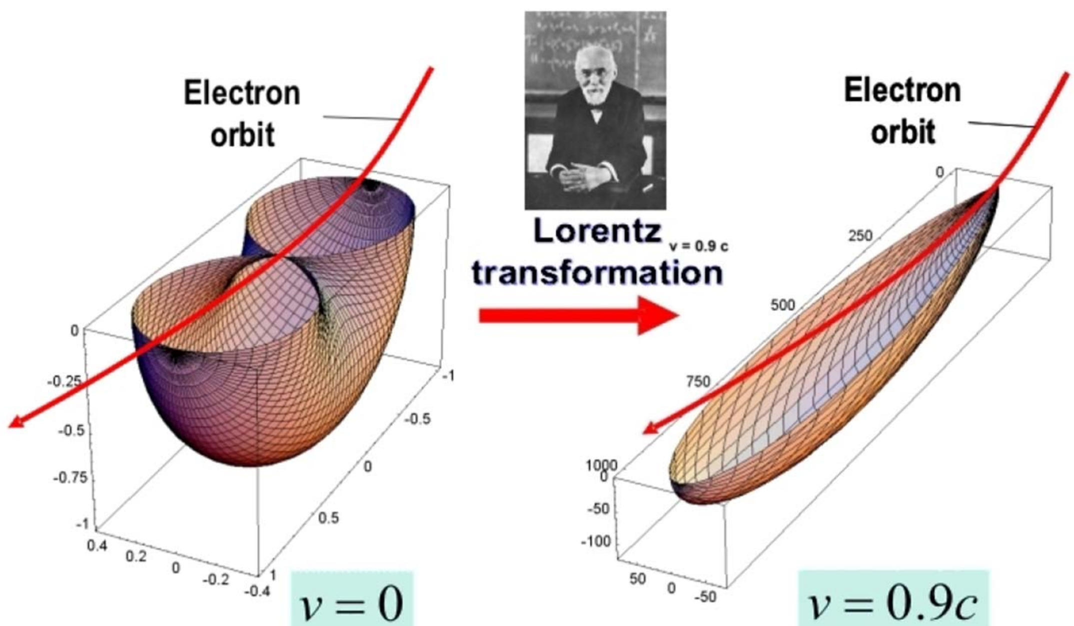
The field generated by a moving charge is given by the Lienard-Wiechert field, with the velocity and acceleration evaluated at the retarded time



$$t - t' = R / c$$

$$\vec{E}(\vec{r},t) = \frac{e(\vec{n}-\vec{\beta}_e)}{\gamma^2 R^2 (1-\vec{n}\bullet\vec{\beta}_e)^3} + \frac{e\vec{n}\times[(\vec{n}-\vec{\beta}_e)\times\dot{\vec{\beta}}_e]}{cR(1-\vec{n}\bullet\vec{\beta}_e)^3}$$

$$\vec{B}(\vec{r},t) = \vec{n}\times\vec{E}(\vec{r},t) \qquad \vec{n} = \vec{R}/R$$



If we call $\theta<<1$ the angle between the velocity and the direction of observation and consider highly relativistic particles the denominator in the radiation term is approximately $(1+\gamma^2\theta^2)/2\gamma^2$

The radiation is peaked in the forward direction within an angle $\theta \le 1/\gamma$

Frequency-angular distribution

In most cases we are interested in the radiation frequency-angular distribution. For one electron this can be written as

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \vec{n}\times(\vec{n}\times\vec{\beta})e^{i\omega(t-\vec{n}\cdot\vec{r}/c)}\,dt\right|^2$$

The integration is extended over the region where the electron is accelerated. The frequency-angular distribution is similar to the Fourier transform of the electron velocity.

For this and other formulas characterizing the radiation an excellent reference is J. D. Jackson Classical Electrodynamics, 3rd edition, John Wiley, N.Y., p. 599 and following, (1998).

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Synchrotron radiation

The spectrum is broadband and is defined by the time of observation of the radiation defined by the angular aperture of the radiation

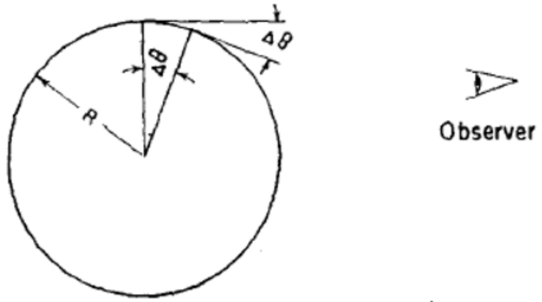
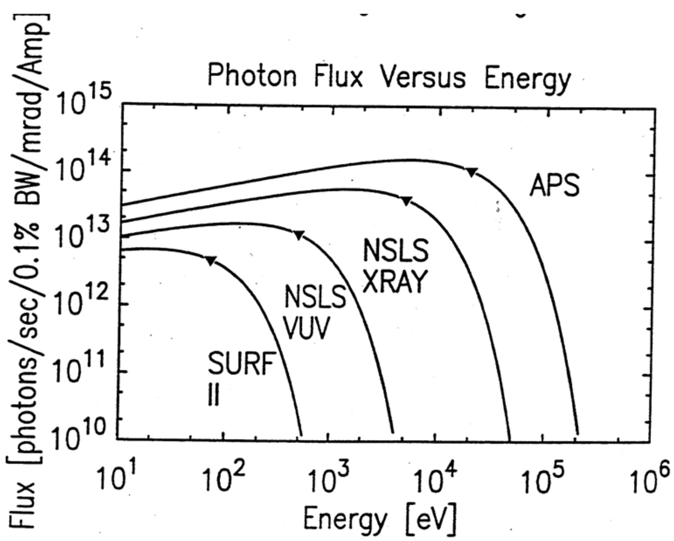


FIG. 2. Illustrates the radiation cone emanating from an electron in a circular orbit.

$$\tau = 2R / \beta\gamma c - 2R\sin(1/\gamma)/c \approx 4R/3\gamma^3 c$$

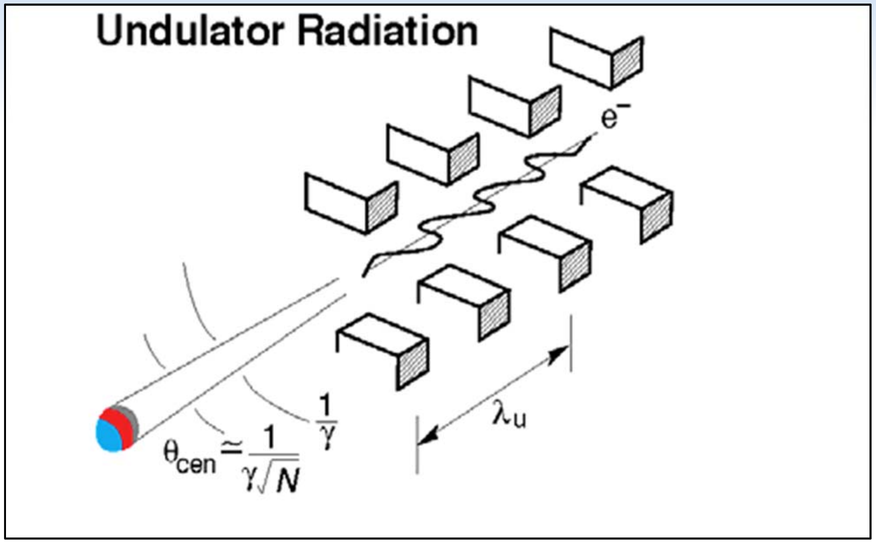


The spectrum falls off exponentially above the critical frequency

$$\omega_c = 3\gamma^3 c / 2R$$

Ref. J. D. Jackson as before.

Undulator magnet



H. Motz , Applications of the radiation from fast electron beams, J. Appl. Phys. 22, 527-535 (1951).

JOURNAL OF APPLIED PHYSICS VOLUME 24, NUMBER 7 JULY, 1953

Experiments on Radiation by Fast Electron Beams*

H. MOTZ, W. THON, AND R. N. WHITEHURST
Microwave Laboratory, Stanford University, Palo Alto, California
(Received October 27, 1952)

Motz proposed the use of a long array of alternating magnetic fields, an undulator magnet, to generate narrow band radiation and have an extended radiation source. The electrons have a sinusoidal trajectory along the axis. Most of the radiation is generated near the point of maximum acceleration.

In his paper Motz evaluates the energy of the radiation pulse and considers the effect of coherent emission, increasing the intensity by a factor as large as the number of electrons in a bunch, N_e .

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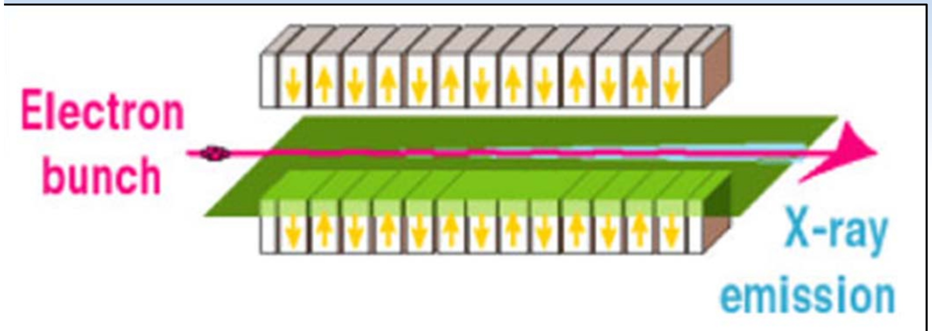
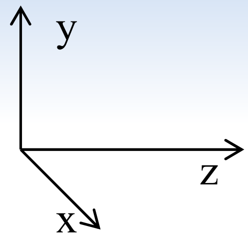
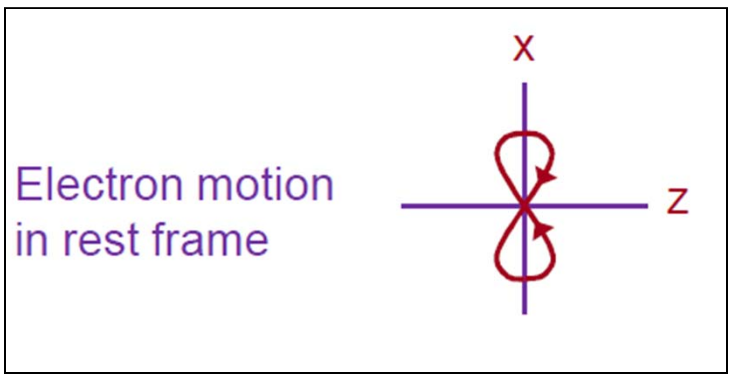
Undulator radiation, single electron





$$B_x = 0$$
$$B_y = B_0 \cosh(k_U y)\cos(k_U z)$$
$$B_z = -B_0 \sinh(k_U y)\sin(k_U z)$$

Near the axis we approximate cosh ~ 1, sinh x ~ x and evaluate the electron velocity.



Electron velocity

$$\beta_x = (K/\gamma)\sin(k_U z)$$
$$\beta_z \simeq 1-(1+K^2/2)(1-\cos(2k_U z))/2\gamma^2$$

$$\boxed{K = \frac{eB_0\lambda_U}{2\pi mc^2}}$$

K is the undulator vector potential normalized to electron rest energy and is an important quantity for the undulator radiation. It is typically of the order of one: K≈ 0.93 B (tesla) λ_U (cm).

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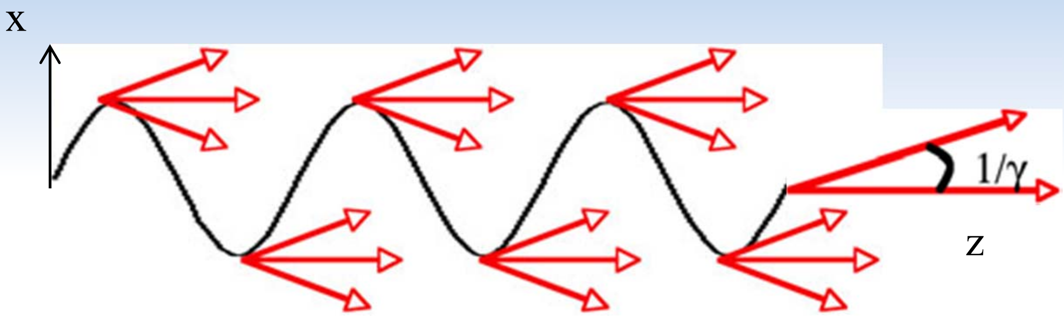
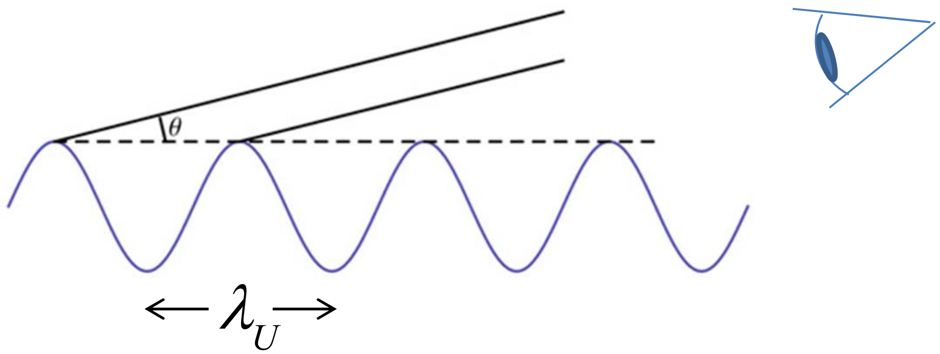
Undulator radiation, resonance condition



Interference effect



Observer

$$\Delta T = \frac{\lambda_U}{\beta_z c} - \frac{\lambda_U}{c}\cos\theta = n\frac{\lambda}{c}$$

$$\boxed{n\lambda = \lambda_U (1 - \beta_z \cos\theta) / \beta_z}$$

Using the expression for the velocity and assuming *θ<<1* we can also write

$$\boxed{\lambda \simeq \frac{\lambda_U (1 + K^2/2 + \gamma^2\theta^2)}{2n\gamma^2} = \frac{\lambda_R}{n}}$$

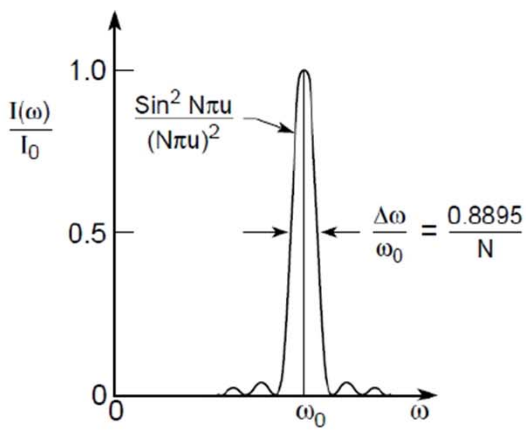
Differential spectrum for undulator radiation

The frequency angular distribution for the undulator radiation is a sum of harmonics. The spectrum on axis, $\theta=0$, contains only odd harmonics, n=1,3,5,… The spectrum for the n-th harmonic is:

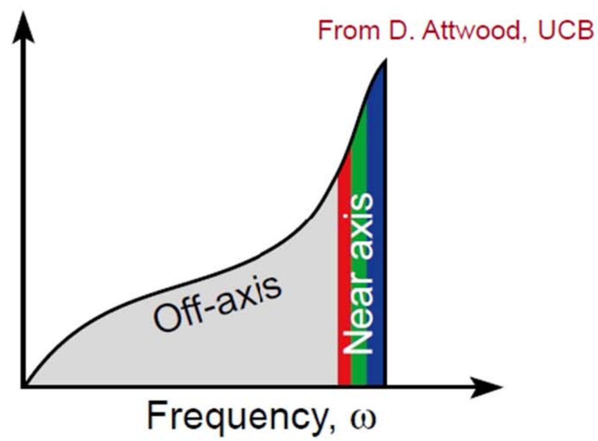
$$\frac{d^2 I}{d\omega d\Omega} = (r_e / c) mc^2 N_U^2 \gamma^2 \frac{K^2}{(1+K^2/2)^2} F_n(K) \left(\frac{\sin(x_n/2)}{x_n/2} \right)^2 ,$$

$$F_n(K) = n^2 \left[J_{(n+1)/2}((nK^2/(4+2K^2)) - J_{(n-1)/2}((nK^2/(4+2K^2)) \right]^2$$

$$x_n = \pi N_U (\omega - n\omega_R)/\omega_R$$



Execution of N electron oscillations produces a transform-limited spectral bandwidth, $\Delta\omega'/\omega' = 1/N$.



The Doppler frequency shift has a strong angle dependence, leading to lower photon energies off-axis.

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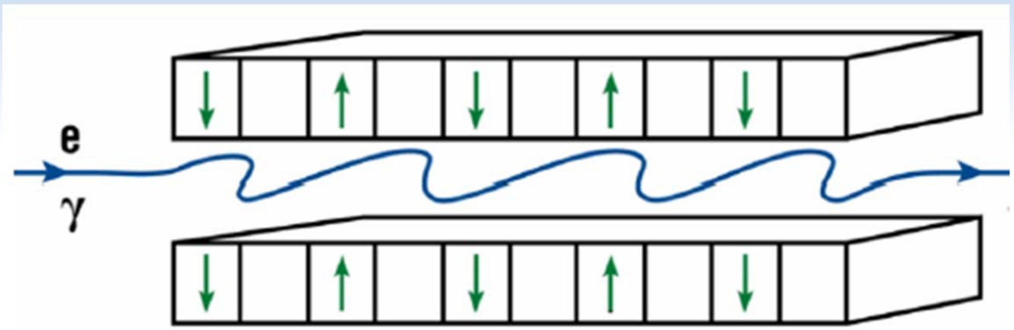
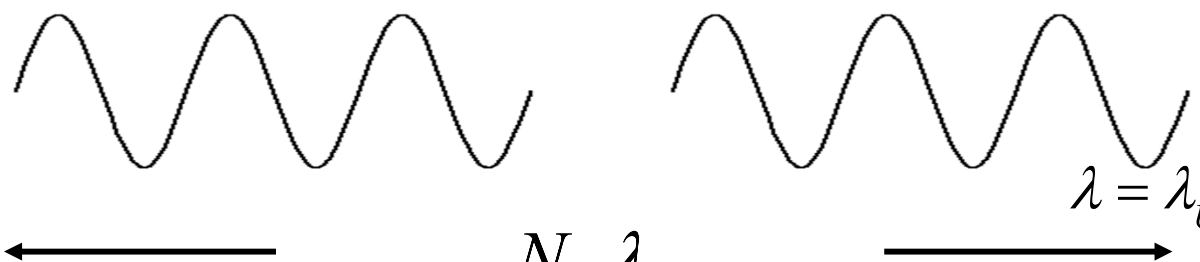
Undulator radiation spectral width



Undulator with N_U periods.

$$\Delta\lambda / \lambda \simeq 1 / N_U$$



$$\lambda = \lambda_U (1 + K^2 / 2 + \gamma^2 \theta^2) / 2\gamma^2$$

$N_U \lambda$

Each electron emits a wave train with N_U cycles

For a case like that of LCLS: $\gamma \approx 3 \times 10^4,\ \lambda_U \approx 3cm,\ K \approx 3,\ N_U \approx 3500$

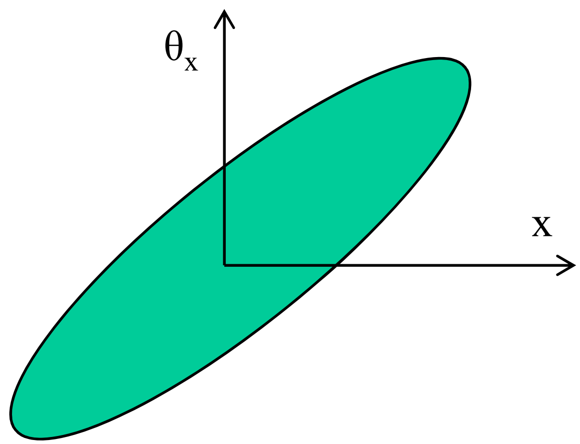
$\lambda = 0.1nm,\ \Delta\omega/\omega \approx 3 \times 10^{-4},\ \lambda N_U = 0.3\mu m (1fs)$

Photon phase space and transverse coherence

Consider the transverse phase space area ($x.\ p_x, y, p_y$) occupied by a photon beam with momentum p. The minimum possible area is

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\Delta y \Delta p_y \geq \hbar / 2$$



Using the angles respect to the direction of propagation $\theta_{x,y} = p_{x,y} / p$ we have*

$$\Delta x \Delta \theta_x \geq \lambda / 4\pi$$

$$\Delta y \Delta \theta_y \geq \lambda / 4\pi$$

The coherent part of the photon beam, the diffraction limited part, is that restricted to the minimum phase space area, $(\lambda/4\pi)^2$.

The concept of phase space area, also called emittance, is useful because it is an invariant when transporting the photon beam through an optical system, including focusing elements. During the transport the phase space area rotates, the beam divergence and spot size change, but their product remains the same.

*Siegman, A.E., Lasers, University Science Books, Sausalito, California (1986).

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Longitudinal coherence

For the longitudinal case we have $\Delta z \Delta p_z \geq \hbar / 2$, or $\Delta E \Delta T \geq \hbar / 2$

For undulator radiation from a single electron we have $\Delta\omega / \omega \simeq 1 / N_U$

Because of the dependence of the emitted radiation wavelength on the emission angle

$$\lambda \simeq \frac{\lambda_U (1 + K^2/2 + \gamma^2\theta^2)}{2\gamma^2}$$

to remain within the line width the emission angle must be limited to

$$\theta_c = \sqrt{\lambda / N_U \lambda_U}$$

If we consider diffraction limited radiation the corresponding source radius is

$$a_c = \sqrt{\lambda \lambda_U N_U} / 4\pi$$

For LCLS $\boldsymbol{\theta_c}$*~ 1* $\boldsymbol{\mu}$*rad,* a_c*~10* μ*m.*

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FEL physics: radiation from one electron

Using the angle-frequency intensity formula and integrating over the line width and a solid angle $\Delta\Omega_c = \pi\theta_c^2$

we obtain the coherent intensity, within the coherent phase space volume and the number of photons in this volume, the degeneracy number for the undulator radiation for a single electron

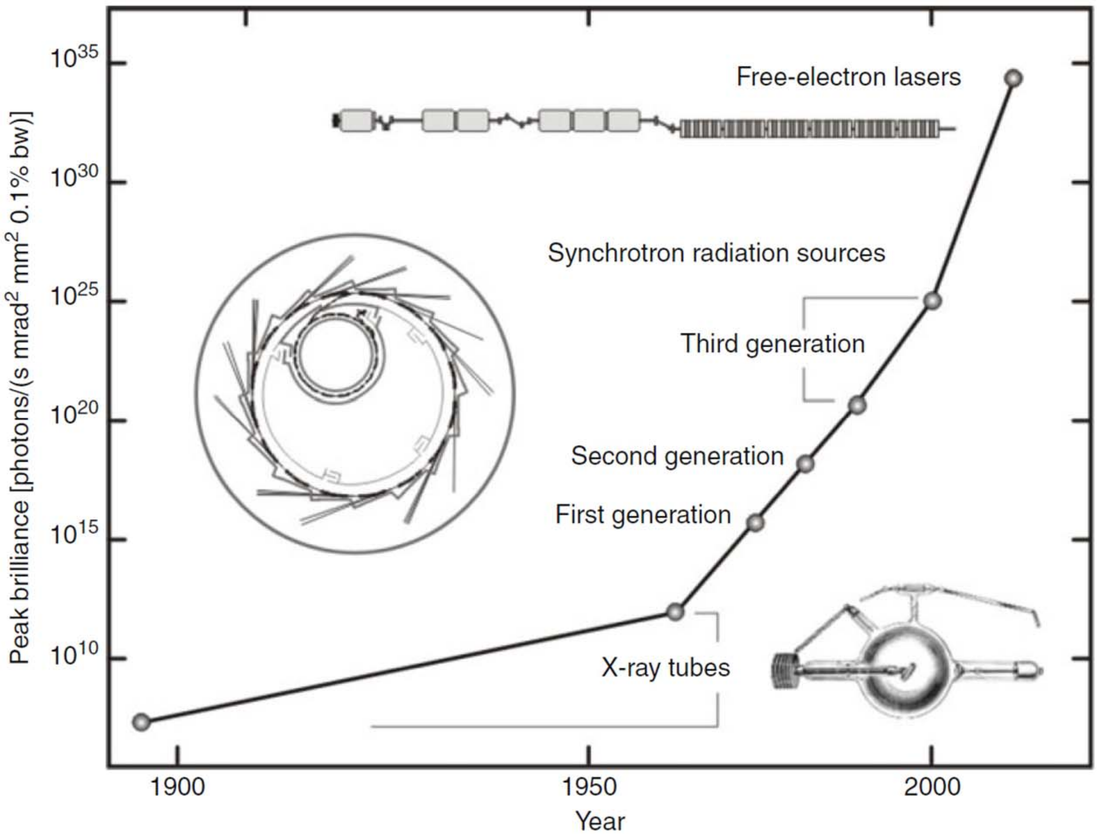
$$N_{ph,c} = \pi\alpha \frac{K^2}{2+K^2} F_1(K)$$

α=1/137 is the fine structure constant.

Since K ~ 1, $N_{ph,c}$ is a small number <<1.
It is remarkable that the number does not depend on wavelength, electron energy or undulator length.

Synchrotron radiation light sources

So far we basically described the physics of synchrotron radiation light sources (originally a spin-off of particle accelerators)



- **1st generation = parasitic facilities (bending magnet radiation)**
  - Initial experiments in the 50s and 60s. SPEAR at SLAC was 1st multi GeV storage ring and SSRP built 5 beamlines
- **2nd generation = dedicated ring for SR generation**
  - Late 70's and 80's. SRS at Daresbury, Photon Factory at KEK, NLSL at BNL
- **3rd generation = optimized for SR generation with insertion devices**
  - ESRF, APS and Spring-8, ALS etc.

Now about 70 light sources worldwide based on storage ring technology (VUV to hard x-rays) and new ones being built and designed

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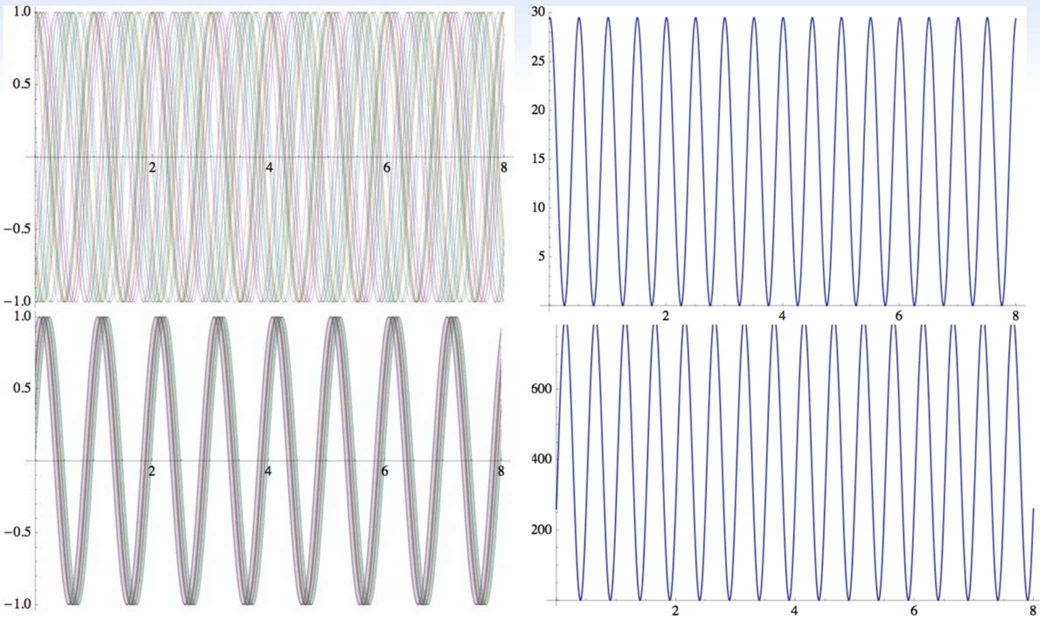
Many electrons.
A picture of their emitted wave trains



Disordered state, the single electron wave trains superimpose with random phases: noise. **Intensity ~ N_e**

Ordered state, all wave trains are in phase. **Intensity ~ N_e^2**

The order parameter is the "Bunching factor", *B*

$$B = \frac{1}{N_e} \sum_{n=1}^{N_e} \exp(i\phi_n)$$

Φ_n is the relative phase of the wave and the electron oscillation. $B=1$ is perfect order.

N_e is about 10^9- 10^{10} , a large gain. But at 1Å we have about 10^3 -10^4 electrons per wavelength. How do we squeeze them in one tenth of the wavelength and have these micro-bunches separated exactly by λ? How do we go from disorder to order? Answer:

FEL

Spread in electron longitudinal positions

The frequency angular distribution for an ensemble of electrons is

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\sum_{k=1}^{N_e}\int_{-\infty}^{\infty}\vec{n}\times(\vec{n}\times\vec{\beta}_k)e^{i\omega(t-\vec{n}\cdot\vec{r}_k/c)}\,dt\right|^2$$

To evaluate the sum we must evaluate how the electrons in the beam are distributed longitudinally on a scale of the radiation wavelength, the energy distribution and the transverse position and angle.

Let us consider first the case when all velocities are equal but there is a spread in the longitudinal electron position.

$$\omega(t-z/c) = \omega\{t(1-\beta_z) - z_0/c\}$$

The integral is the same for all electrons except for the phase factor $\exp(2\pi i z_0/\lambda)$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^2} N_e^2 \left|B(\omega)\right|^2 \left|\int_0^{L_U/c}\vec{\beta}_T e^{i\omega(1-\beta_z)t}\,dt\right|^2$$

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Bunching factor

As an example for the case of a Gaussian bunch with rms length σ_0

$$N_e^2 |B|^2 = N_e + N_e(N_e - 1)e^{-\omega^2 \sigma_0^2 / c^2}$$

We have an intensity proportional to N_e^2 if $\sigma_0 \ll \lambda$.

At the nanometer or sub-nanometer wavelength there is no simple way to control the longitudinal electron position to obtain directly a large bunching factor. The generation of electrons at the cathode is a random process and the initial electron distribution is dominated by Schottky noise. In this situation we have*

$$\langle B \rangle = 0 \qquad\qquad N_e^2 \langle |B|^2 \rangle = N_e$$

The intensity fluctuates and is proportional to the electron number.

*Goodman J.W., Statistical Optics Wiley (1985)

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Spread in electron velocities

Consider the expression of the transverse velocity that we used before

$$\beta_x = (K/\gamma)\sin(k_U z)$$
$$\beta_y = 0$$

This can only be true for an electron arriving at the undulator entrance with the transverse velocity equal to zero. In this ideal case all electrons follow the same trajectory around the undulator axis. In the transverse electron phase space the beam would be represented by a point and would have zero phase space volume. This is in contradiction to the uncertainty principle.

To avoid being unphysical we must have a spread of transverse position and velocity at the undulator entrance (i.e. consider beams with finite emittance)

$$\beta_x = (K/\gamma)\sin(k_U z) + \beta_{x0}$$
$$\beta_y = \beta_{y0}$$

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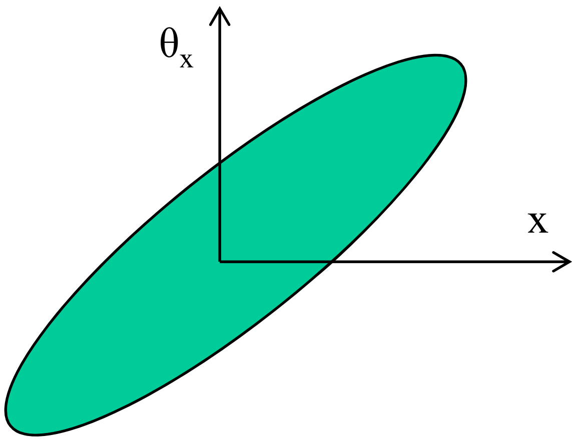
Electron transverse phase space

Consider the transverse phase space area ($x.\ p_x,\ y,\ p_y$) occupied by an electron beam with longitudinal momentum p. The minimum possible area is



$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\Delta y \Delta p_y \geq \hbar / 2$$

Using the angles respect to the direction of propagation $\theta_{x,y} = p_{x,y} / p$ we have

$$\Delta x \Delta \theta_x \geq \lambda_c / 4\pi$$

$$\Delta y \Delta \theta_y \geq \lambda_c / 4\pi$$

The limit is very small and is not of any practical importance for an intense electron beam like those we want to use for an FEL. In fact for efficient generation of coherent radiation we want the beam phase space area to be smaller than that of the photon beam. In electron beam language we call the emittance $\varepsilon_{x,y}$ the phase space area in the $(x,\ \theta_x)$ plane or the $(y,\ \theta_y)$ plane and normalized emittance the same quantities in the position-momentum planes.

$$\varepsilon_{x,y} \leq \lambda / 4\pi$$

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Effect of angular position and momentum spread on undulator radiation

Consider the formula for the radiation wavelength $\lambda \simeq \dfrac{\lambda_U(1+K^2/2+\gamma^2\theta^2)}{2\gamma^2}$

The effect of energy, transverse angle-position spread generates a spread in radiation wavelength. The transverse position changes the values of K because the magnetic field increases when moving off the middle plane, the $\cosh(k_U y)$ in the transverse magnetic field.

$$\frac{\Delta\lambda}{\lambda} = -\frac{2}{1+K^2/2}\frac{\Delta\gamma}{\gamma} + \frac{K^2}{1+K^2/2}\frac{\Delta K}{K} + \frac{\gamma^2\theta^2}{1+K^2/2}$$

The last two terms are an effective energy spread due to the transverse position and angle distribution. As in any optical system we can focus the beam to reduce the transverse size, or to decrease the angles. However we cannot reduce the two together because of the conservation of transverse phase space area. Focusing for electron oscillations around the zero order orbit is characterized by the oscillation (betatron) wave number β_B. The beam transverse phase space area is given by the beam transverse emittance $\sigma_{x,y}^2 = \varepsilon_{x,y}\beta_{B,x,y},\ \sigma_{\theta_x,\theta_y}^2 = \varepsilon_{x,y}/\beta_{B,x,y}$

$$\left.\frac{\Delta\gamma}{\gamma}\right|_{eff} = \frac{\varepsilon}{2}\left(K^2 k_U^2 \beta_B + \frac{\gamma^2}{\beta_B}\right)$$

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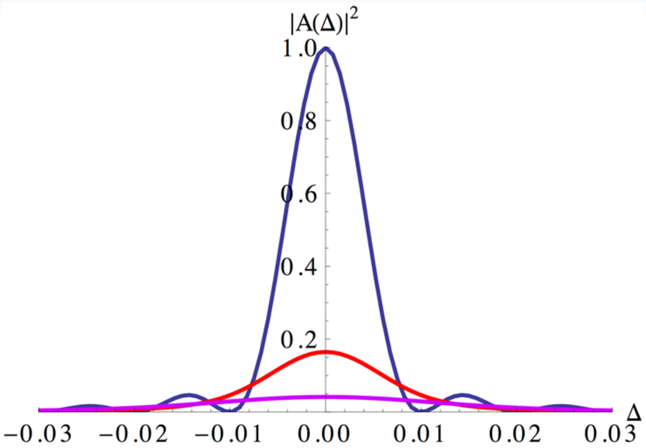
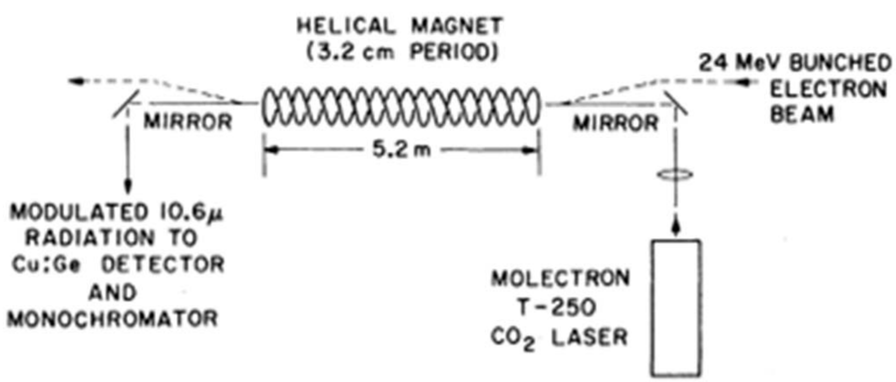
Effect of energy spread on radiation intensity on axis



Relative intensity on axis for an undulator with 100 periods and a Gaussian energy distribution with an rms energy spread $1/10N_U$, blue line, $1/N_U$, red line, $2/N_U$, purple line. To avoid losses the relative energy spread, including emittance effects, must be smaller than the line width.

Stimulated undulator radiation : FEL

The next step is to analyze stimulated undulator radiation, that is introduce an electromagnetic wave at a frequency near the undulator radiation frequency co-propagating with the electron beam along the undulator axis.



Madey's amplifier experiment. Gain 7%.

Madey, J.M.J. 1971. "Stimulated emission of bremsstrahlung in a periodic magnetic field," J. Appl. Phys. 42: 1906-1913.
Elias, L. et al. Observation of stimulated emission of radiation by relativistic electrons in a spatially periodic transverse magnetic field,. Phys. Rev. Lett. 36: 717-720 (1976).

A remarkable result of Madey's quantum theory is that the laser gain does not depend on Planck's constant. The FEL is a classical system. Another result is that the gain scaling with wavelength is not favorable to reaching X-rays.

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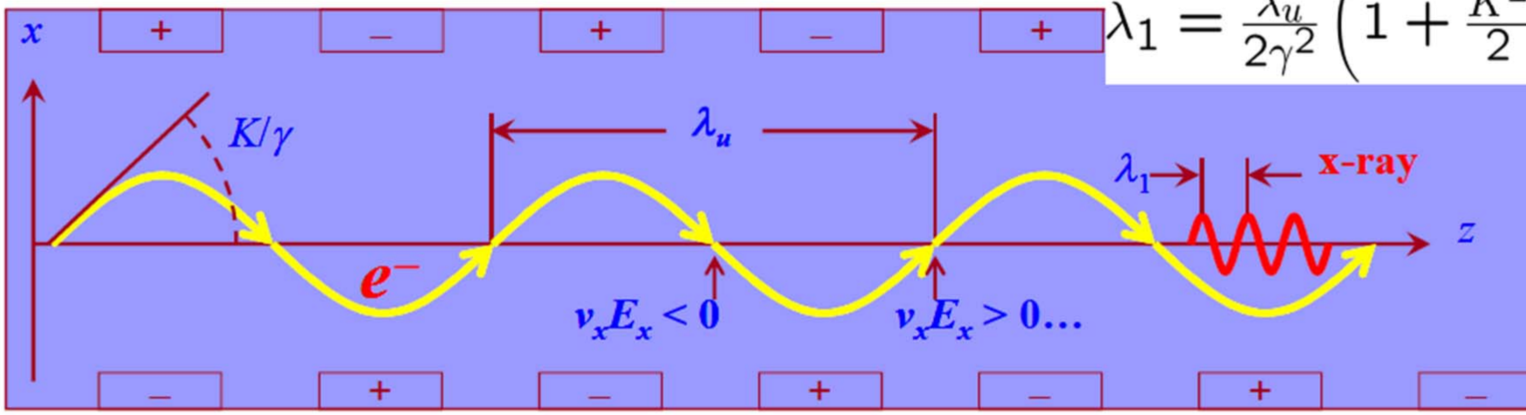
FEL principle of operation

- Electrons **slip** behind EM wave by λ_1 per undulator period (λ_u)

$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$



- Due to sustained interaction, some electrons lose energy, while others gain ➔ **energy modulation at λ_1**

- e^- losing energy slow down, and e^- gaining energy catch up ➔ **density modulation at λ_1 (microbunching)**

- Microbunched beam radiates coherently at λ_1, enhancing the process ➔ **exponential growth of radiation power**

Bonifacio, R., C. Pellegrini and L. Narducci. Collective instabilities and high-gain regime in a free electron laser. Opt. Commun. 50: 373-378 (1984). Kondratenko and E.L. Saldin, Dokl. Aka. Nauk SSSR 249, 843 (1979)

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Include radiation electric field

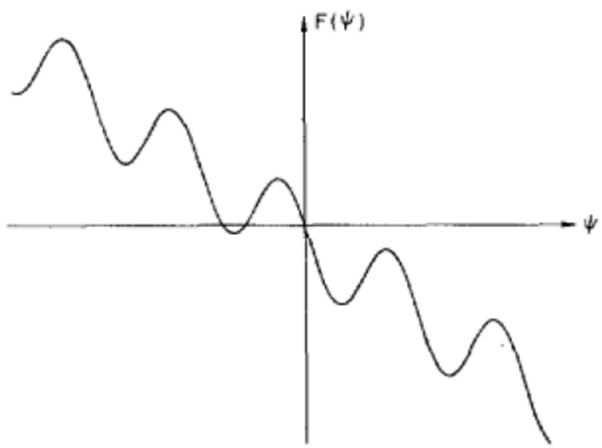
Electron-EM wave energy exchange equation $$\boxed{\frac{d\gamma}{dt} = \frac{e}{mc^2}\mathbf{E}\cdot\mathbf{v}}$$

Assume an electric field: $E_x = E_0 \cos(ks - \omega t + \varphi)$

$$\frac{\mathrm{dW}}{dt} = -\mathrm{e}E_0 \cos(ks - \omega t + \varphi) \cdot \frac{cK}{\gamma} \sin(k_u s) = -\frac{ecE_0 K}{2\gamma}[sin\Psi_+ - sin\Psi_-]$$

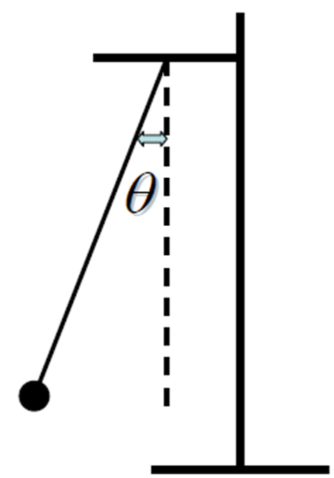
where $\Psi_{\pm} = (k \pm k_u)s - \omega t + \varphi$

FEL particle dynamics equations for planar undulator



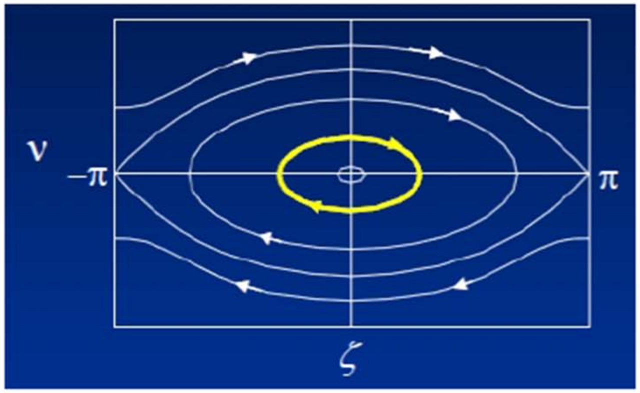
$$\frac{d\gamma}{dz} = \frac{kKK_l\, JJ\ \sin\psi}{2\gamma}$$

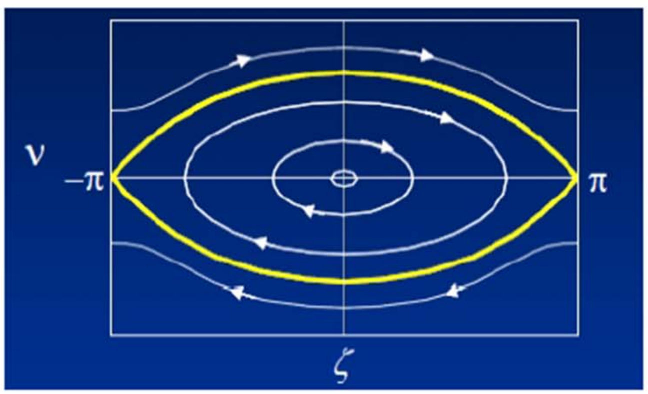
$$\frac{d\psi}{dz} = k_w - k\frac{1 + \frac{K^2}{2}}{2\gamma^2}$$



Ponderomotive potential

Pendulum equation described the transfer of energy to and from the electrons as a function of the pondermotive phase





Small amplitude oscillations

$\Psi = \Psi_0 \sin \Omega s$ where

$$\Omega^2 = \frac{K\, k_u a_L k_L}{\gamma^2}$$

with $a_L = eE_0/mc\omega_L$ (sometimes refered to as a_s)

Large amplitude oscillations

oscillation frequency decreases as a function of amplitude

Bucket height which depends on the undulator strength and the laser field

$$\eta_{max} = \sqrt{\frac{K\, a_L}{\sqrt{2(1+K^2/2)}}}$$

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Field equations

Narrow bandwidth resonant process $\frac{\Delta\omega}{\omega} \approx \rho \approx 10^{-3}$

Slow wave approximation

$$E(z,r,t) = \tilde{E}(z,r,t)\exp\left(i(kz - \omega t)\right)$$

Slowly varying function which satisfies $\left|\frac{\partial^2 \tilde{E}}{\partial z^2}\right| << k^2 \tilde{E}$ - $\left|\frac{\partial^2 \tilde{E}}{\partial t^2}\right| << \omega^2 \tilde{E}$

Maxwell equations can be written for the slow term $a_r e^{i\varphi_r} \propto \tilde{E}/k$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A}(\vec{x},t) = -\frac{4\pi}{c}\vec{J}(\vec{x},t) \Rightarrow \left[2ik_r\frac{\partial}{\partial z} + \nabla_\perp^2\right]a_r(r,z)e^{i\varphi_r} = -J(r,z)\left\langle\frac{a_w e^{-i\vartheta}}{\gamma}\right\rangle$$

- The slowly varing field equation may be solved with a PDE solver *(as in TDA3D – GENESIS – GINGER)*
- Or by projection on Gauss or Laguerre modes: SDE* expansion *(as e.g. in MEDUSA)*

* *P.Sprangle, A. Ting, C.M. Tang Phys. Rev A 36 (1987)*

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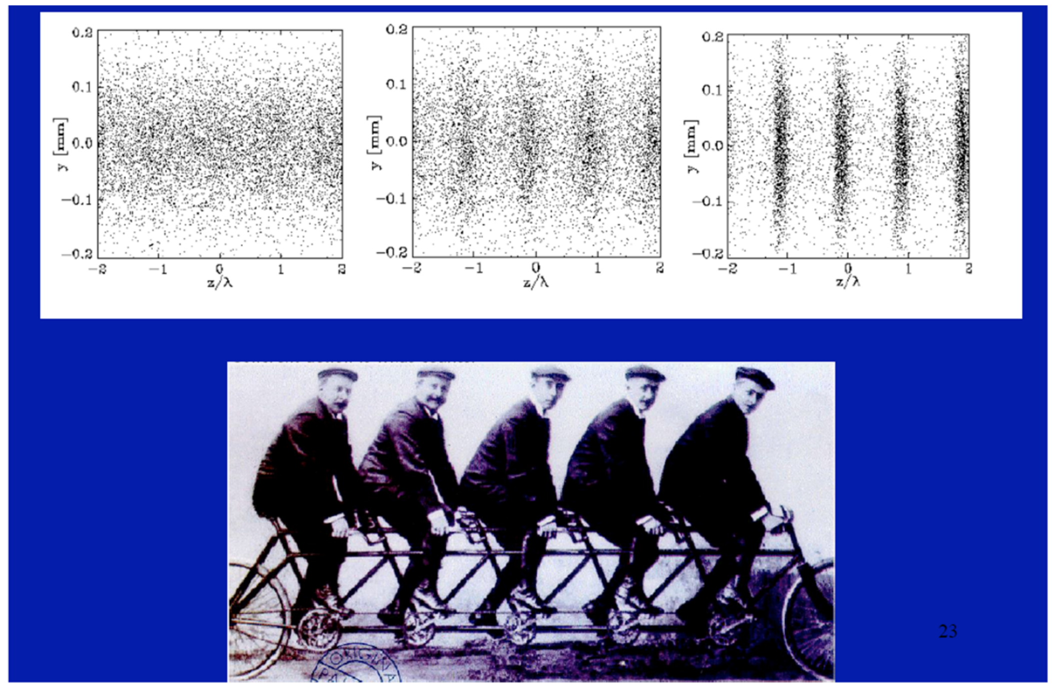
FEL Collective Instability, a self-organization effect

1. e-beam+undulator +EM field (can also be the initial spontaneous radiation)-> electron energy modulation, scale λ;



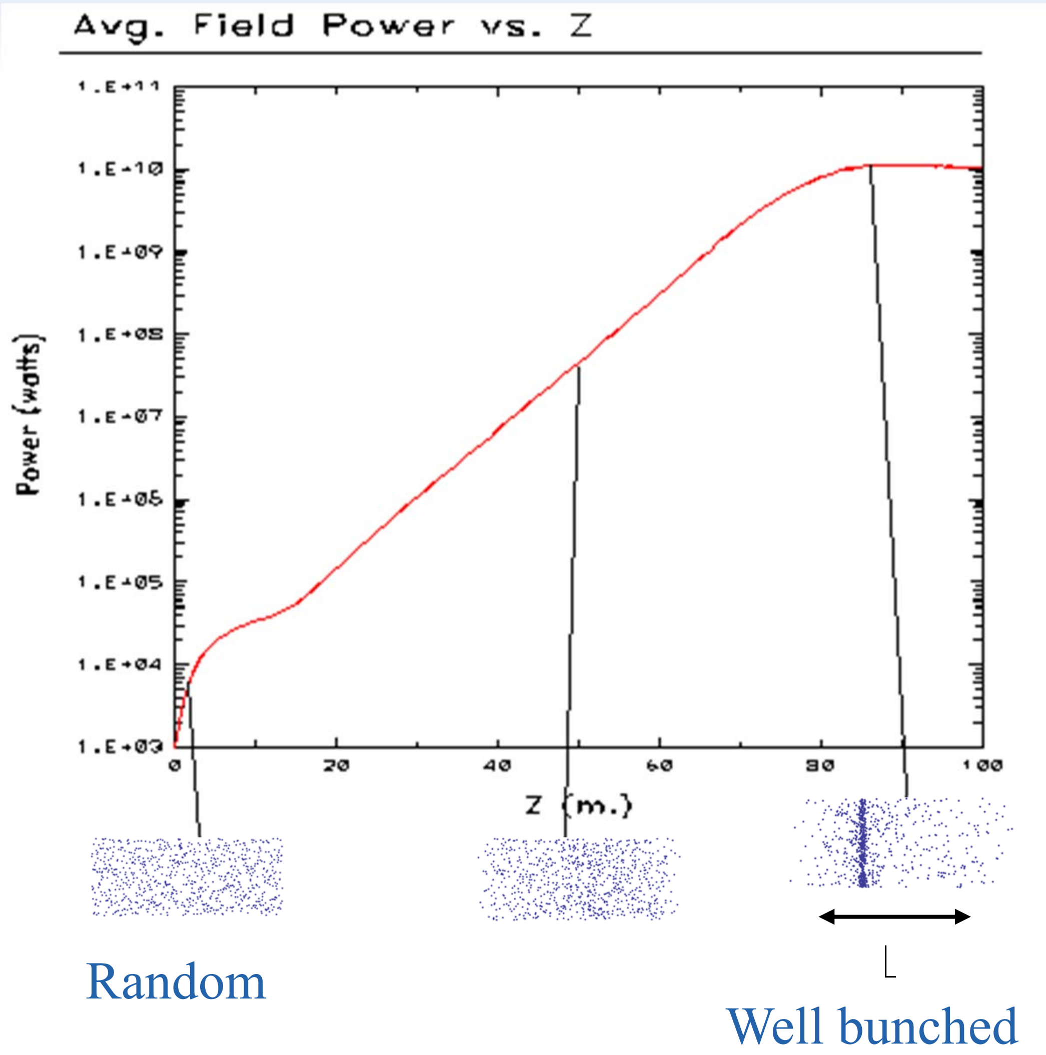
2. energy modulation + undulator -> electron bunching, scale λ;

3. larger bunching factor B->higher EM field intensity ->go back to step 1

SASE: a beam self-organization effect.

The self organization effect can start from the initial noise at the undulator radiation frequency in the electron beam longitudinal distribution, the same that gives the spontaneous radiation. This is a SASE FEL. The instability produces an ordered distribution in the beam, similar to a 1-d relativistic crystal.

Evolution of power and longitudinal beam density along the undulator from spontaneous radiation to FEL amplified radiation.



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The FEL equations

The FEL dynamics can be described using two characteristics length, the gain and the cooperation lengths, in a "universal form", where only the FEL parameter plays a role. This description is "wavelength independent". Once the theory is verified at one wavelength it works at all wavelengths, as long as we stay in the classical regime.

The FEL 1-D universal equations

1. Phase $$\frac{\partial \Phi_l}{\partial \bar{z}} = \eta_l$$

1. Energy $$\frac{\partial \eta_l}{\partial \bar{z}} = (\hat{\alpha} e^{i\Phi_l} + cc)$$

2. Field $$\left(\frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}_1}\right)\hat{\alpha} = -\left\langle e^{-i\Phi_l} \right\rangle$$

$$\eta = (\gamma - \gamma_R)/\rho\gamma_R$$

is the energy deviation in units of the FEL parameter.

$\bar{z}$, $\bar{z}_1$ are the coordinates along the undulator and the electron bunch measured in units of gain length and cooperation length.

The phase change is zero when the beam energy is the resonant energy.

R. Bonifacio, et al., Phys. Rev. Lett. 73, 70 (1994)

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The FEL equations

The system has an equilibrium point for zero field, energy equal to resonant energy, no spread, zero bunching factor. Near the equilibrium point it can be reduced to 3 equations for the field, the bunching and the energy deviation $P = \left\langle \eta e^{-i\Phi} \right\rangle$

$$\frac{\partial}{\partial \bar{z}} B = i\delta - iP \qquad \frac{\partial}{\partial \bar{z}} P = \hat{\alpha}. \qquad \left(\frac{\partial}{\partial \bar{z}} + \frac{\partial}{\partial \bar{z}_1} \right) \hat{\alpha} = -B$$

δ, the detuning, is the relative difference between the average beam energy and the resonant energy in units of the FEL parameter.

The system is easily analyzed and one can show that there is an unstable solution giving an exponential growth if $\delta<1.9$. The gain length is the growth rate. The unstable solution can grow from an external field, seeded FEL, the initial random value of the bunching factor, SASE FEL, or an energy deviation from the resonant energy. The last is usually a small contribution.

FEL Collective Instability: main characteristics

All key characteristics are given by one universal FEL parameter: $\rho = \left(\frac{K}{4}\frac{\omega_P}{\omega_U}\right)^{2/3}$ $\omega_P = (4\pi r_e c^2 n_e / \gamma^3)^{1/2}$ $\omega_U = 2\pi c / \lambda_U$

- Gain Length: $L_G=\lambda_U/4\pi\rho$,
- Saturation: $P\sim\rho\, I_{beam}\, E$
- Saturation length: $L_{sat}\sim 10 L_G \sim\lambda_U/\rho$
- Line width: $\sim\rho$

Number of photons/electron at saturation: $N_{ph}\sim \rho QE/E_{ph}$. For E_{ph}=10keV, E=15 GeV, Q=0.1 nC (6x10^8 electrons), $\rho=10^{-3}$, $N_{ph}\sim 10^{12}$, a gain of 5 orders of magnitude respect to the spontaneous radiation.

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FEL Collective Instability: electron beam quality conditions

The exponential growth occurs if

$\sigma_E < \rho$ Cold electron beam

$\varepsilon \approx \lambda/4\pi$ Electron-photon phase-space matching

$Z_R/L_G > 1$ Diffraction losses from the beam less than the gain

The beam Rayleigh range is $Z_R = \pi a^2 / \lambda$, where a is the beam transverse radius.

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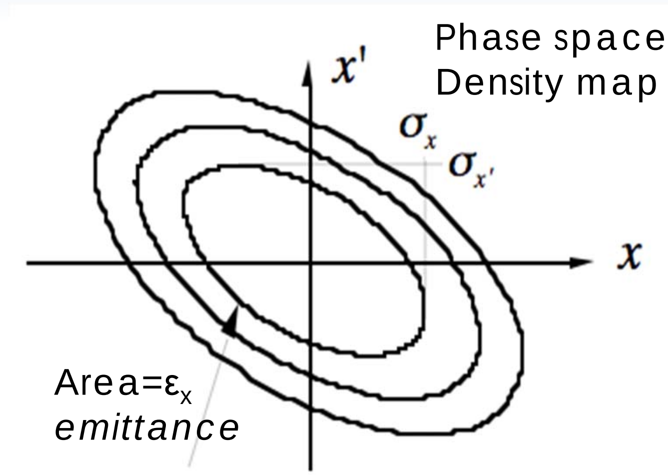
Essential ingredient of FEL: high brightness electron beam

FEL Pierce parameter $\rho \sim \frac{\text{radiation power}}{\text{beam power}}$

peak current

$$\rho = \frac{1}{4\gamma_0}\left[\frac{I}{I_A}\frac{\lambda_u^2 K^2 [JJ]^2}{\pi^2 \epsilon_x \beta_x}\right]^{1/3}$$

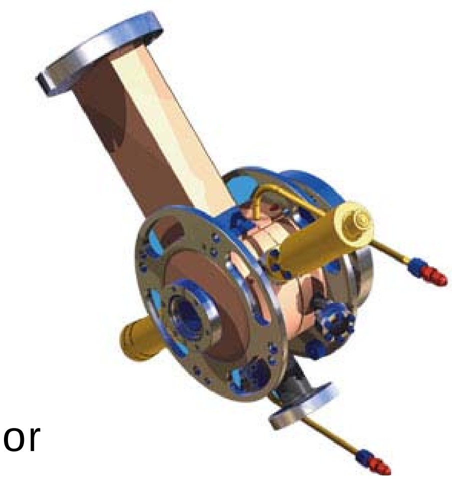
17 kA emittance beta function



- High phase space density (**cold, focusable, intense**)
- Measure: high brightness

$$B_e = \frac{2I}{\varepsilon_x^2}$$

- Space-charge (*plasma*) effects strong in high brightness beams



The secret:
RF photoinjector
(also UCLA)

Classical and quantum theories

The first FEL theory by John Madey was a quantum theory of stimulated bremsstrahlung, in the limit of a small change of the radiation field amplitude. However the final result, the gain, did not contain Planck's constant.

The following theoretical work was mostly classical.

The parameter that defines the regime is the electron recoil parameter

$$q = \hbar\omega / mc^2\gamma\rho$$

If q<<1 we are in the classical regime. The electron can emit many photons before its energy is outside the FEL gain bandwidth.

If q~1 we are in the quantum regime, the number of photons/electron is ~1, compared to 1000 for the classical case at 1A.

We consider only the classical regime.

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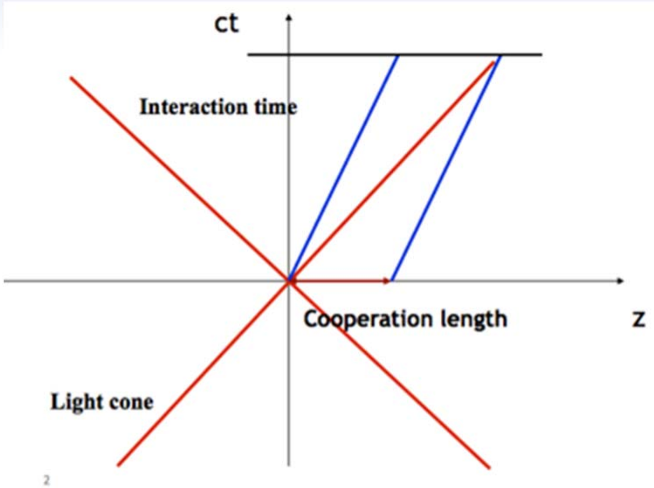
FEL Collective Instability: causality, slippage, cooperation length



- The radiation propagates faster than the electron, "slips" by λ per undulator period. Electrons communicate with those in front if their separation is less than the total slippage $S=N_U \lambda_U$.

Cooperation length: slippage in one gain length:

$$L_c = \lambda / 4\pi\rho$$

- The local intensity in a SASE radiation pulse is proportional to the initial random bunching within a cooperation length, leading to the formation of "spikes", with independent intensity.
- Number of "spikes" in an X-ray pulse: bunch length/$2\pi L_c$.
  (R. Bonifacio, et al., Phys. Rev. Lett. 73, 70 (1994)).

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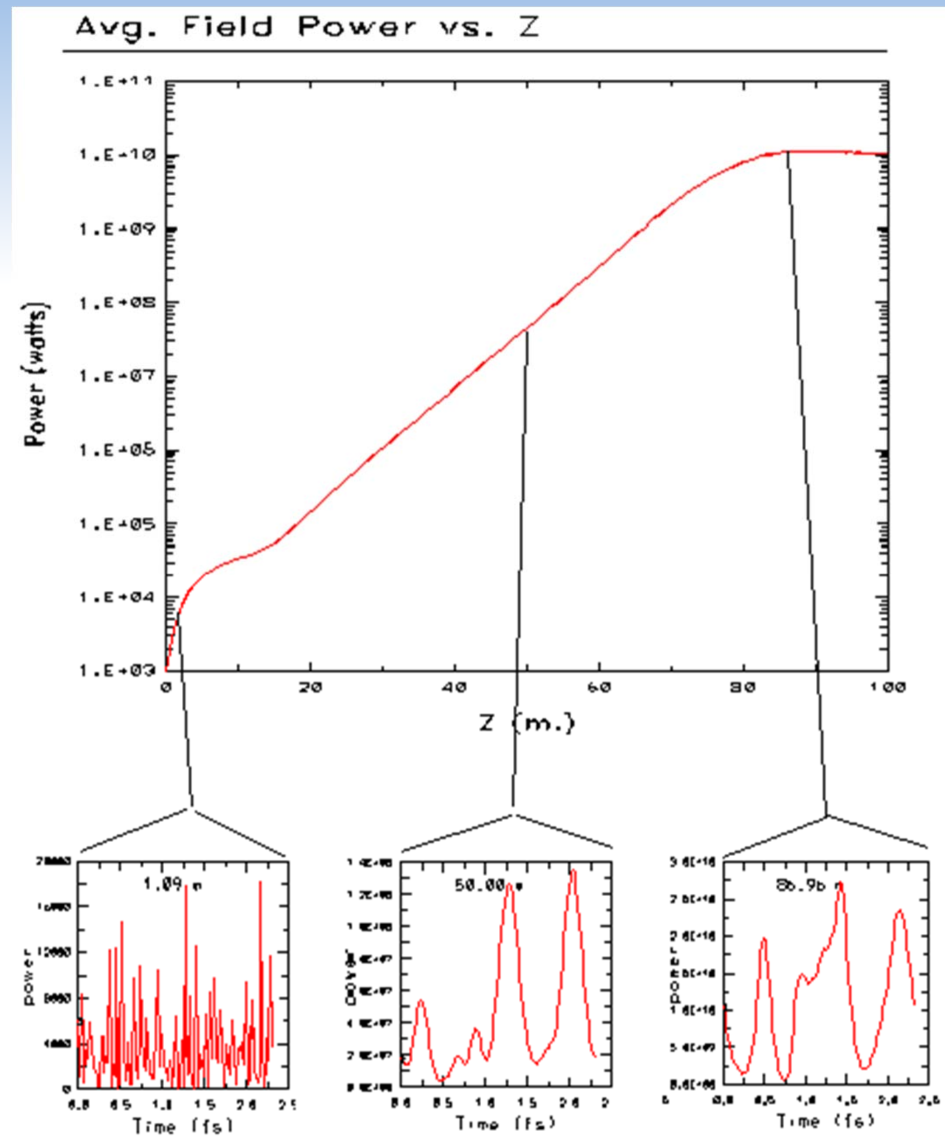
The spiky nature of a SASE-FEL

LCLS: L_c=40 nm
Full spike length is ~ 0.24 μm or 0.8 fs.
The number of spikes depends on the bunch length.



A SASE FEL is not transform limited, its spectral width corresponds to the spike length, not to bunch length.

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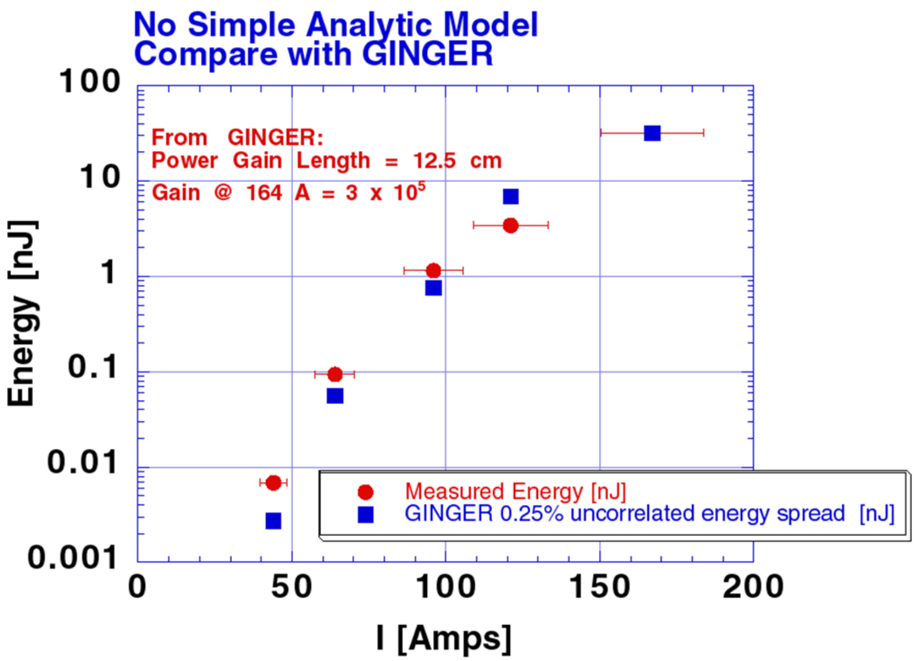
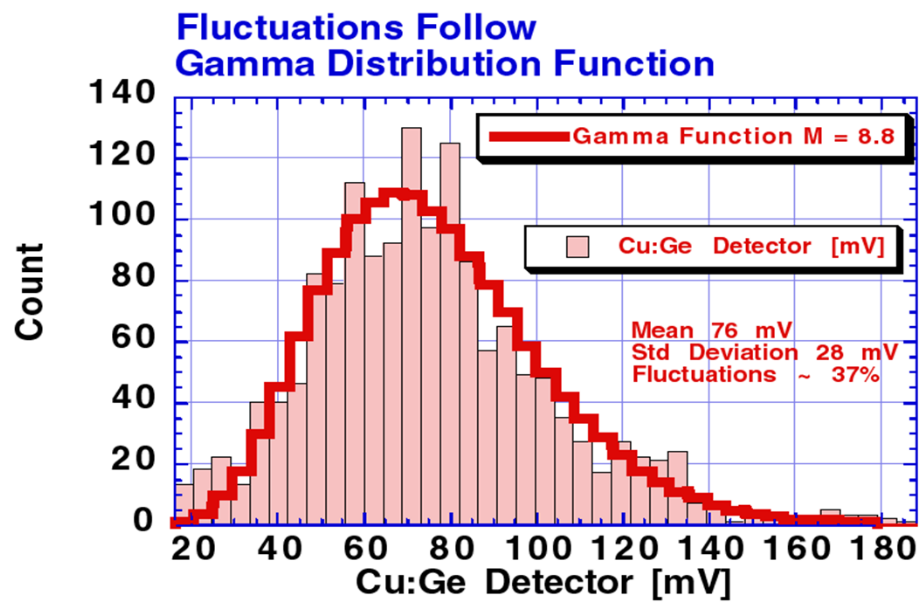
Experimental verifications of theory

UCLA/Kurchatov/LANL/SSRL . Gain of 3×10^5 at 12 µm.

Demonstration of fluctuations and spikes.

Agreement with theory. M. Hogan et al. Phys. Rev. Lett. 81, 4897 (1998).





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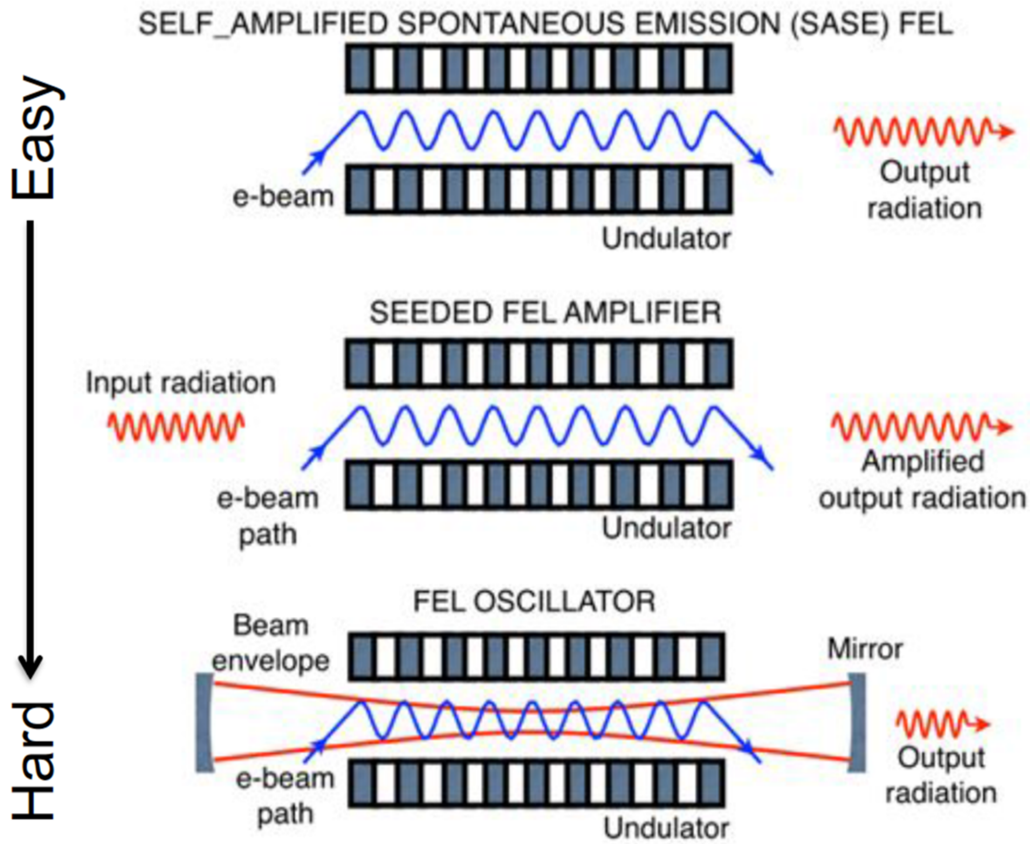
Options for free electron lasers



LCLS USA
SACLA Japan
FLASH Germany
XFEL Germany
SWISSFEL Switzerland
PAL FEL Korea

Hard X-rays,
$E_{photon} \geq 5$keV

FERMI Italy
FLASH-II Germany
LCLS

Soft X-rays,
$E_{photon} \leq 1$ keV

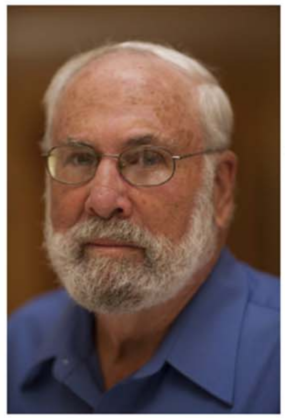
Proposal ANL USA

K.-J. Kim et al.

C. Pellegrini, A 4 to 0.1 nm FEL Based on the SLAC Linac,
Workshop on Fourth Generation Light Sources, February, 1992

Claudio Pellegrini

Herman Winick

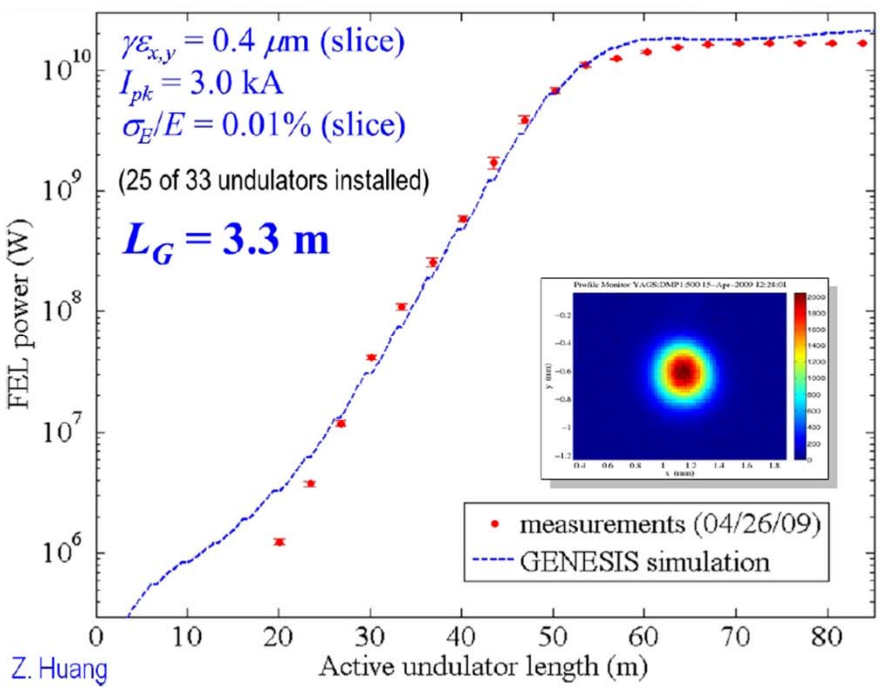


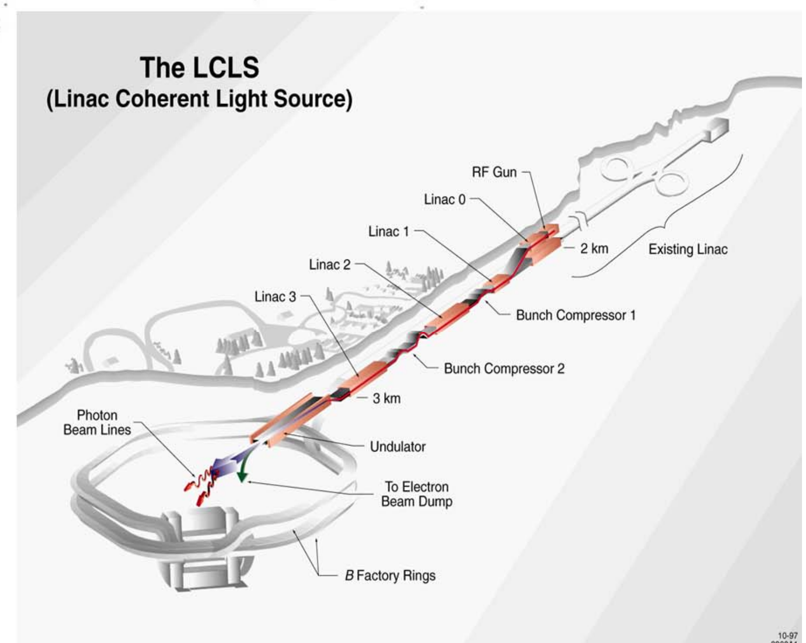
Herman Winick's Study Group

SHORT WAVELENGTH FELs at SLAC - STUDY GROUP

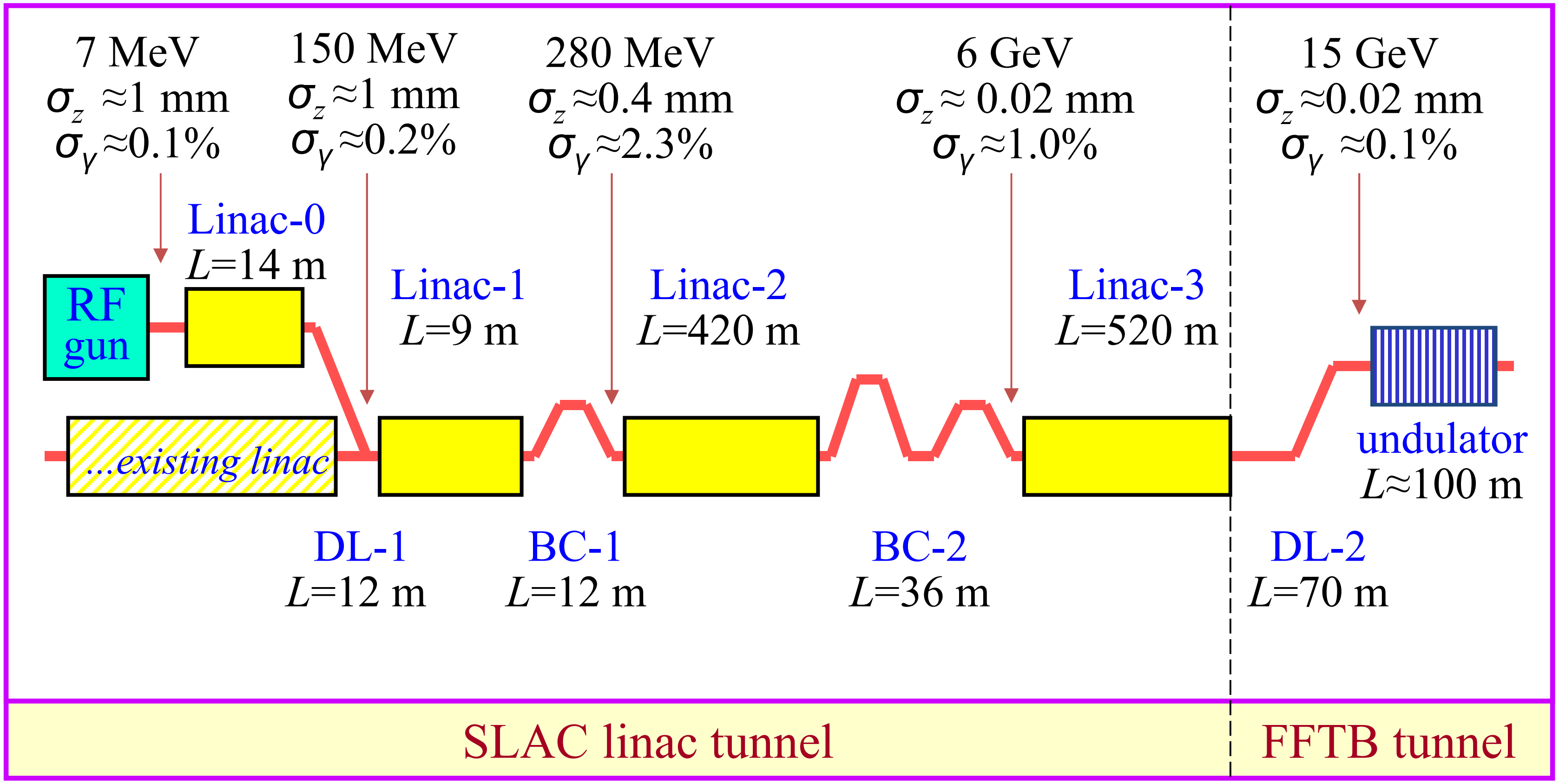
SOURCE	SCIENTIFIC CASE
Karl Bane	Art Bienenstock
Jeff Corbett	Keith Hodgson
Max Cornacchia	Janos Kirz (SUNY-Stony Brook)
Klaus Halbach (LBL)	Piero Pianetta
Albert Hofmann	Steve Rothman (UCSF)
Kwang-je Kim (LBL)	Brian Stephenson (IBM)
Phil Morton	
Heinz-Dieter Nuhn	
Claudio Pellegrini (UCLA)	
Tor Raubenheimer	
John Seeman	
Roman Tatchyn	
Herman Winick	

Engaged Bjorn Wiik and Gerd Materlik during sabbaticals at SLAC





LCLS schematic



Hard X-rays FELs					
	LCLS	SACLA	European XFEL	Korean X-FEL	Swiss X-FEL
Electron energy range, GeV	2.15-15.9	5.2-8.45	8.5-17.5	4-10	2.1-5.8
Wavelength range, nm	.11-4.4	0.063-0.275	0.04-5.1	0.1-0.6	0.1-7
X-ray pulse energy, mJ,	1-3, $0.1<\lambda<$ 1.5nm	0.2-0.4 for $0.08<\lambda<$ 0.27 nm	0.67-8.5, $0.04<\lambda<5.1$ nm	0.81-1, $0.1<\lambda<0.6$ nm	0.5-1.3, $0.1<\lambda<7$ nm
Pulse duration, rms, fs	5-250, $0.1<\lambda<$ 1.5nm	4.3, $0.08<\lambda<$ 0.275 nm	1.68-107, $0.04<\lambda<5.1$ nm	8.6-26, $0.1<\lambda<0.6$ nm	2-20, $0.1<\lambda<7$ nm
Line width, rms, %, SASE	0.5-0.1, $0.1<\lambda<$ 1.5nm	0.11-0.37, $0.08<\lambda<0.275$ nm	0.02-0.25, $0.04<\lambda<5.1$ nm	0.15-0.18, $0.1<\lambda<0.6$ nm	0.06-0.4, $0.1<\lambda<7$ nm
Line width, rms, %, seeded	0.01-0.005*, $0.1<\lambda<$ 1.5nm	0.01*-0.003*, $0.08<\lambda<0.275$ nm	0.04-0.005, $0.04<\lambda<5.1$ nm	0.002-0.002, $0.1<\lambda<0.6$ nm	0.01-0.002, $0.1<\lambda<7$ nm

* Values based on simulations

Notes

1. Pulse energy hundreds of μJ to few mJ.
2. Line width in SASE mode about 10^{-3}, order of magnitude of the FEL parameter ρ.
3. Pulse durations from a few to about 100 fs.
4. About 10^3 photons/electron at about 1 A (10^{-2} for spontaneous radiation).

Soft X-rays FEL characteristics

	FLASH	Fermi
Electron beam energy, GeV	0.37-1.25	1.5
Wavelength range, nm	45-4.2	65-10
X-ray pulse energy, mJ	0.2 @ λ_{Max} 0.5 @ λ_{min}	0.03 @ λ_{Max} 0.01 @ λ_{min}
Pulse duration, rms, fs	15-100 @ λ_{Max} 15-100 @ λ_{min}	<40 @ λ_{Max} n.a.
Line width, rms,%, SASE	0.2 @ λ_{Max} 0.5 @ λ_{min}	
Line width, rms, %, seeded		0.06 @ λ_{Max} 0.03 @ λ_{min}

Flash operates as SASE FEL. An upgrade, Flash II, is being built as an HHG laser seeded FEL.

Fermi is presently the only X-ray FEL operated as an HGHG system. The driving laser wavelength is about 260nm. It has two FELs: FEL 1 with $20<\lambda<65$ nm, and FEL 2 with $4<\lambda<10$.

Pulse energy < 1 mJ,
pulse duration about 15 to 100 fs,
line width about 10^{-3} or larger if SASE.

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Frontiers of XFEL research

The existing X-rays FELs are doing quite well and have reached or went above the design parameters. The main areas where their performance can be improved are:

1. Improve the longitudinal coherence of SASE-FELs
2. Increase the energy transfer from the electron to the photon beam, presently about 0.1%, to a > 1 percent
3. Manipulate the X-ray spectrum, generate 2 or more lines with variable delay or simultaneous for pump probe or stimulated emission experiments
4. Generate atto-second pulses
5. Use high gradient accelerators. Compact X-FELs

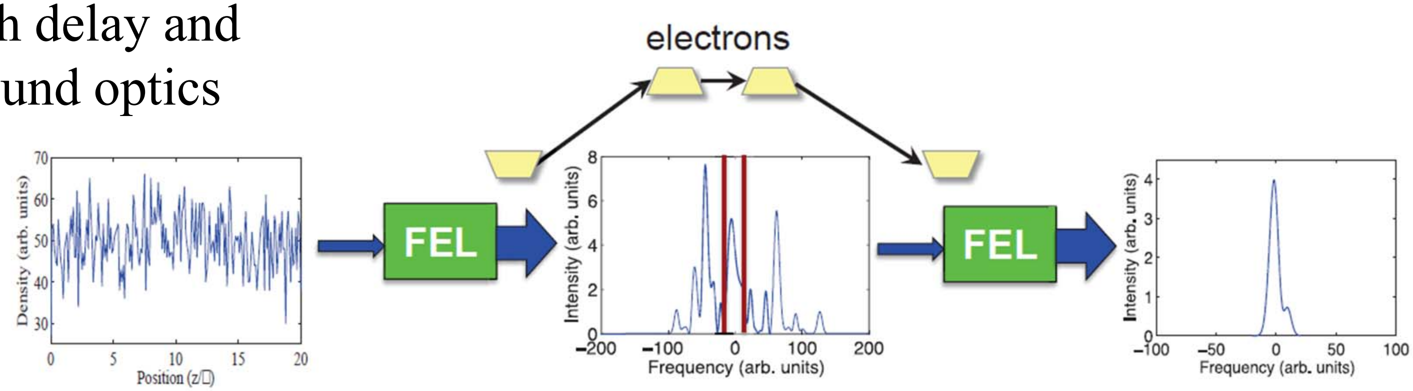
Improved longitudinal coherence

Self-seeding:

Requires two components

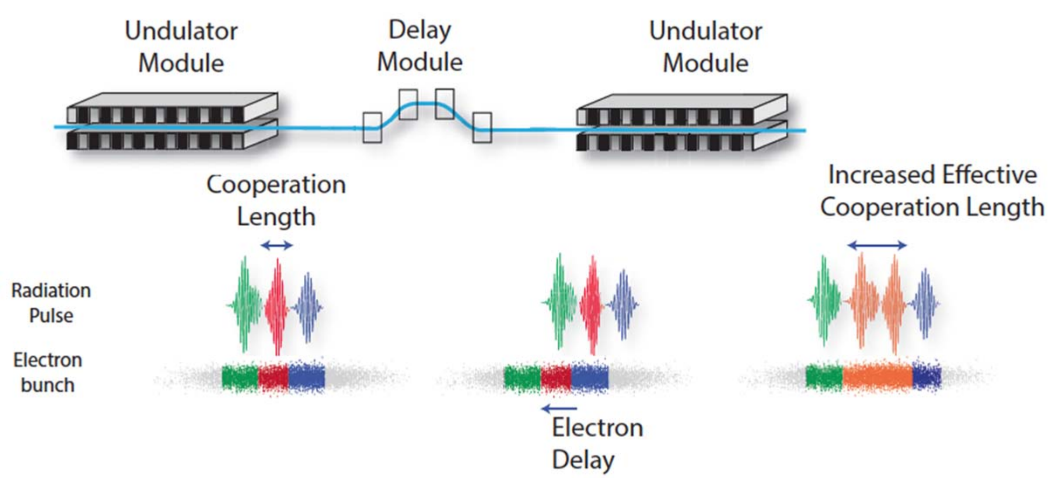
a) Monochromator

b) Chicane to match delay and steer e-beam around optics



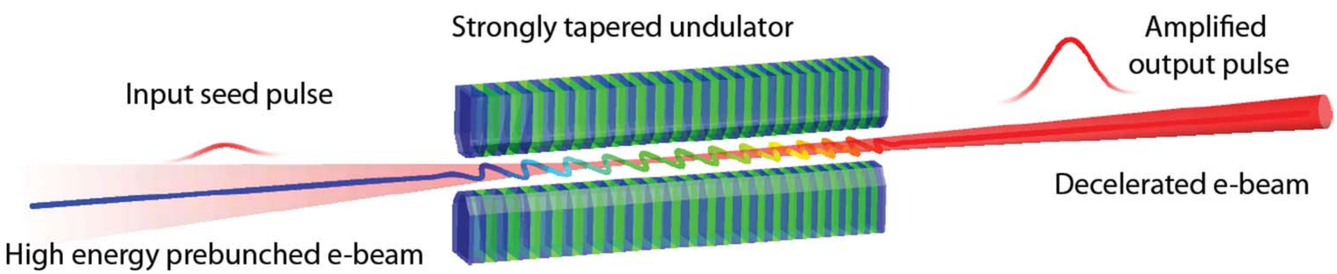
ISASE:

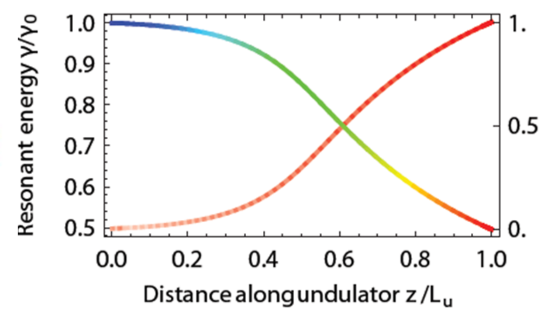
Delay line introduces longer coherence legnth



Saturation and tapering

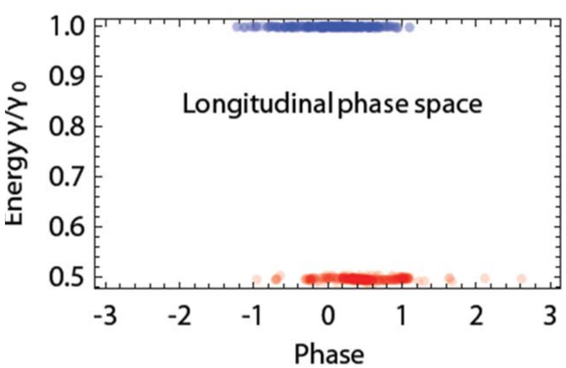
- FEL saturation occurs when particles lose energy and fall out of resonance. Efficiency limited to ρ.
- Solution : Taper undulator parameter to keep particle in resonance and sustain energy exchange.

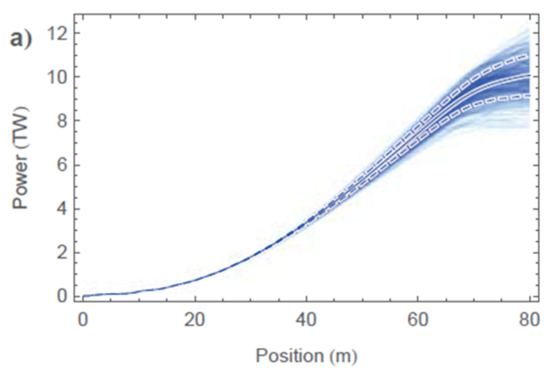


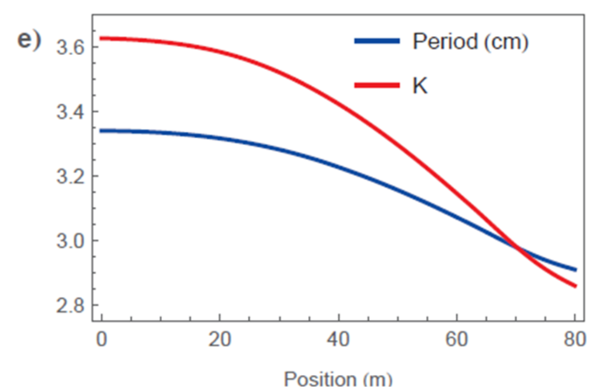


TW X-FEL

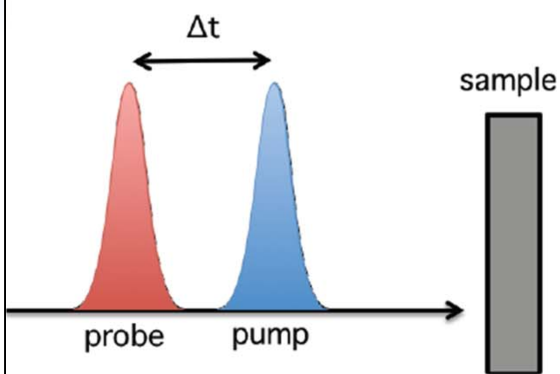
TESSA optimization
arXiv:**1410.6787**







Two colors



$$\lambda_{1,2} = \lambda_w \frac{1+K_{1,2}^2}{2\gamma^2}$$

$$\lambda_{1,2} = \lambda_w \frac{1+K^2}{2\gamma_{1,2}^2}$$

2 pulses with

-tunable energy difference

-tunable arrival time

Many applications!

- x-ray pump/x-ray probe
- 2 color diffraction imaging

PRL 110, 134801 (2013) PHYSICAL REVIEW LETTERS week ending 29 MARCH 2013

Experimental Demonstration of Femtosecond Two-Color X-Ray Free-Electron Lasers

A. A. Lutman, R. Coffee, Y. Ding,* Z. Huang, J. Krzywinski, T. Maxwell, M. Messerschmidt, and H.-D. Nuhn
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(Received 13 December 2012; published 25 March 2013)

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Multicolor Operation and Spectral Control in a Gain-Modulated X-Ray Free-Electron Laser

A. Marinelli,[1,*] A. A. Lutman,[1] J. Wu,[1] Y. Ding,[1] J. Krzywinski,[1] H.-D. Nuhn,[1] Y. Feng,[1] R. N. Coffee,[1] and C. Pellegrini[2,1]

ARTICLE
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Two-colour hard X-ray free-electron laser with wide tunability

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Chirped Seeded Free-Electron Lasers: Self-Standing Light Sources for Two-Color Pump-Probe Experiments

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ARTICLE
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Two-colour pump-probe experiments with a twin-pulse-seed extreme ultraviolet free-electron laser

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Observation of Time-Domain Modulation of Free-Electron-Laser Pulses by Multipeaked Electron-Energy Spectrum

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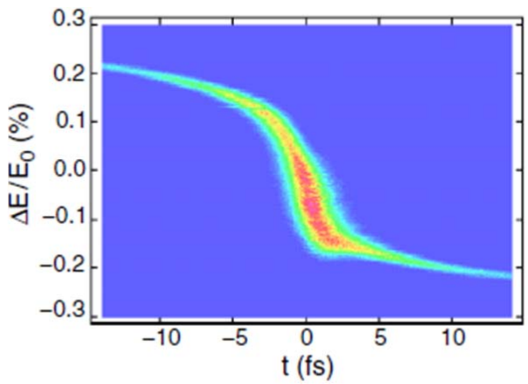
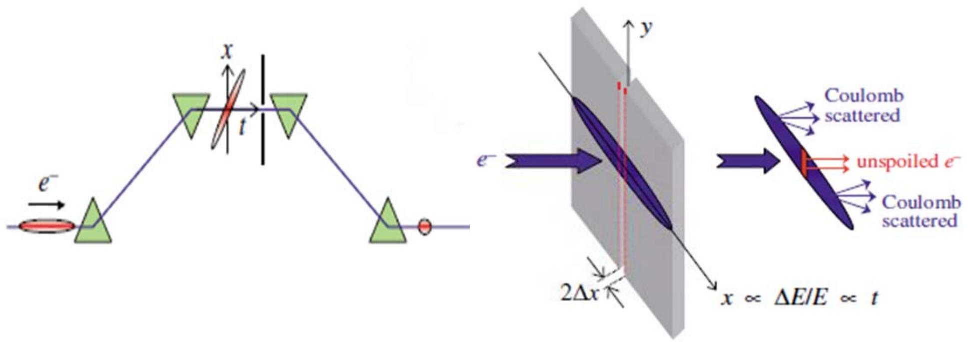
Attosecond x-ray pulses in FELs

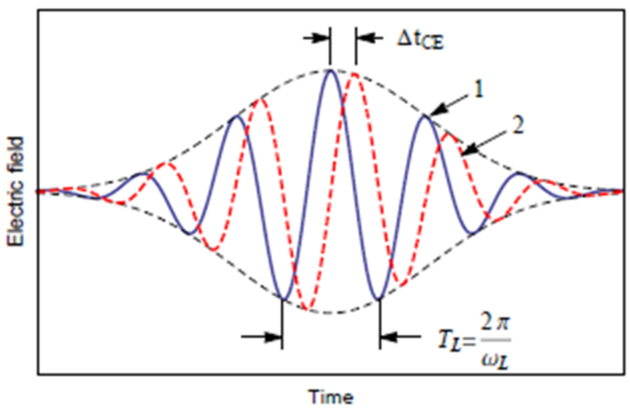
- **Faster pulses are needed to explore the dynamics of fast events**
- **Compress a low charge beam**



- **Slotted foil in a chicane**



- **Using a few-cycle laser**



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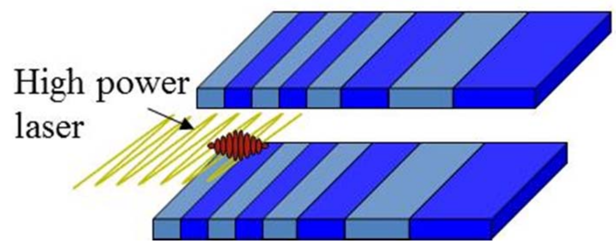
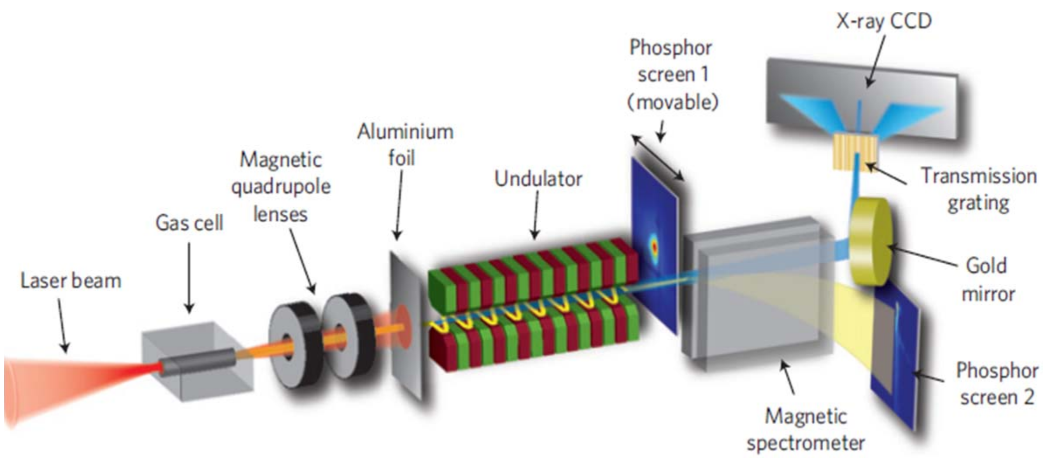
Compact XFEL

- *5th generation light source.*
- Bridge gap between advanced accelerator concepts and FEL physics
- Likely to be the first application of HEP-born high gradient advanced accelerators.

- Plasma, Dielectric, IFEL. Many candidates.
- High brightness beams required

IFEL





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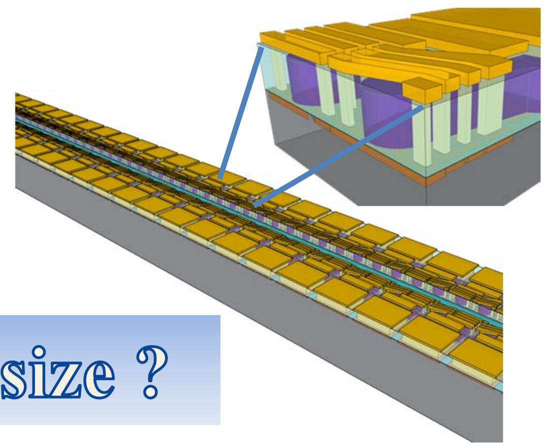
Micro-undulators



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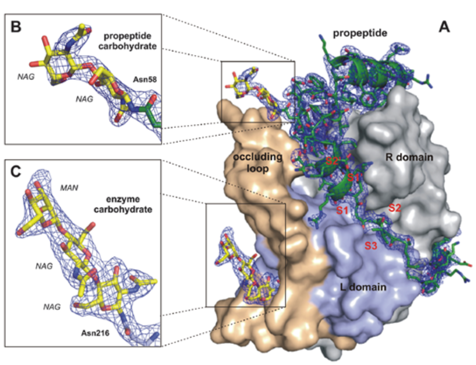
Can we shrink XFEL to < 10 m size ?

Conclusions

X-ray FELs give us an unprecedented view of the structure and dynamics of matter at the angstrom, fs time scale. The flexibility of the system allows to optimize the X-ray pulse intensity, time duration and spectral properties to the experiment being done.



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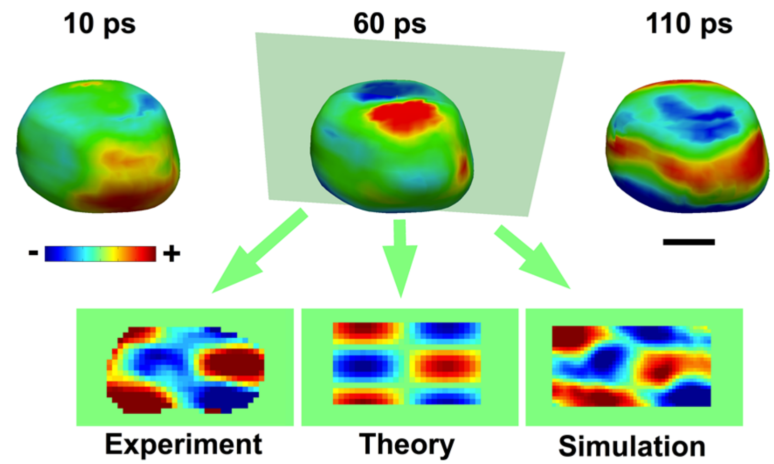

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